Affine Geometry:

There exists a set of objects called Points (Things)

There exists another set of objects called Lines (Bunches). Each Line (Bunch) is a set that includes some Points (Things).

If there is a Point (Thing) and a Line (Bunch) then either the Point is in/on the Line or the Point is not in/on the Line.

1. For every two distinct Points (Things) there is exactly one Line (Bunch) that contains both of them
2. Any time there is a Line (Bunch), and a Point (Thing) that is not in/on that Line, then there is another Line that contains the Point, and does not intersect with the first Line.

Projective Geometry:

There exists a set of objects called Points (Things)

There exists another set of objects called Lines (Bunches). Each Line (Bunch) is a set that includes some Points (Things).

If there is a Point (Thing) and a Line (Bunch) then either the Point is in/on the Line or the Point is not in/on the Line.

1. For every two distinct Points (Things) there is exactly one Line (Bunch) that contains both of them
2. Any time there are two distinct Lines (Bunches), then there is exactly one Point (Thing) that is in both Lines.

For each of these examples,

a) decide if it is a unique-line geometry (satisfies axiom 1)
b) decide if it is an affine geometry, a projective geometry or neither (which axiom 2, if any, does it satisfy?)

1. Usual plane geometry, where Points are points and Lines are infinite straight lines.

2. The usual plane, where the Points are Points but the Lines are circles.

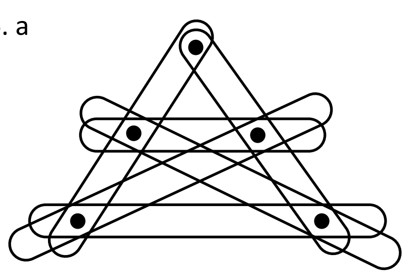
3. The upper half of the x-y plane where the Points are points with (x,y) coordinates, where y>0, and the Lines come in two types: if it is a vertical half-line (equation x=a), then it is a Line, and if it is the upper half of a circle whose center is on the x-axis, then it is a Line.

4. The unit sphere, where Points are points on the sphere, and Lines are great circles (circles that are the intersection of the sphere and a plan that passes through the center of the sphere).

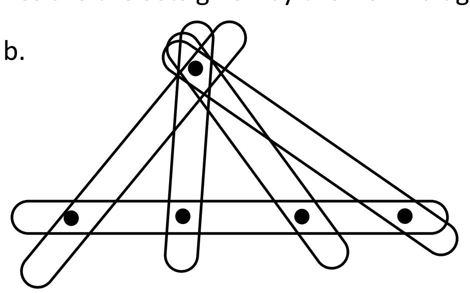
5. The unit sphere, where Points are pairs of antipodal points, and Lines are great circles.

In the diagrams below, where Points are the dots, and Lines are the sets given by the Venn-diagram loops:

6. a



b.



Turn in: Prove (or attempt to prove) that the x-y plane where Points are ordered pairs of real numbers: (x,y) and Lines are sets of points that satisfy an equation of the form ax+by=c (where a, b, and c are real numbers, and a and b are not both 0) is an Affine Geometry.