Thm 37

Given $\triangle ABC$ and $\triangle DEF$ such that

$\overline{AB} \cong \overline{DE}$ (1)

$\overline{BC} \cong \overline{EF}$ (2)

$\overline{AC} \cong \overline{DF}$ (3)

By Ax 3, there is an isometry f, such that

$f(A) = D$ (4)

$f(C) \in \overrightarrow{DF}$ (5)

$f(B) \in$ side of $\overleftrightarrow{DF}$ without E. Let $f(B) = B'$.

By Thm 26 and lines (3, 4, 5) $f(C) = F$ (7)

By isometry $\overline{AB} \cong \overline{DB'}$ (8)

By substitution (1, 8) $\overline{DB'} \cong \overline{DE}$ (9)

By isometry $\overline{BC} \cong \overline{B'F}$ (10)

By substitution (2, 10), $\overline{B'F} \cong \overline{EF}$ (11)

$\overline{DF} \cong \overline{DF}$ (12)

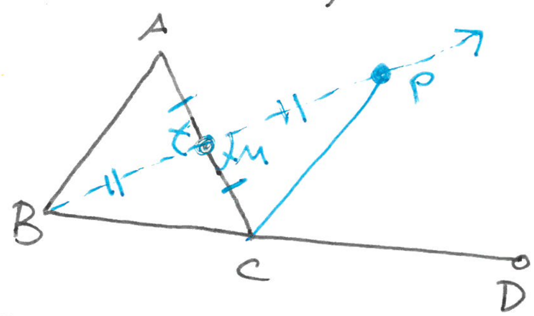
By thm 36 (9, 11, 12) $\triangle DEF \cong \triangle DB'F$

By isometry $\triangle ABC \cong \triangle DB'F$

By thm 17, $\triangle ABC \cong \triangle DEF$

Proof of 40 a

Given $\triangle ABC$, extend $\overline{BC}$ to $\overline{BD}$



By thm 39, $\overline{AC}$ has a midpoint, M

so $\overline{AM} \cong \overline{MC}$ (1)

Construct* $\overrightarrow{BM}$ *or "consider"

By Axiom 5, there exists a point P on $\overrightarrow{BM}$

such that $\underline{BP} = 2\,\underline{BM}$

and $\underline{BM} + \underline{MP} = \underline{BP}$ (defn. of Ray & segment)

so $\overline{BM} \cong \overline{MP}$ (2)

Consider* $\triangle ABM$, $\triangle CPM$ *or construct

By thm 29 $\angle AMB \cong \angle CMP$ (3)

By SAS and (1, 2, 3)

$\triangle ABM \cong \triangle CPM$

so (CPCTC) $\angle BAM \cong \angle PCM$

By Ax. 4 $m\angle ACD = m\angle PCM + m\angle PCD$

so $m\angle ACD > m\angle PCM = m\angle BAM = m\angle BAC$

$\therefore$ $m\angle ACD > m\angle BAC$ $\square$

optional: $\Leftarrow$ $P \in \overline{BM}$ $\Rightarrow$

$\underline{BP} + \underline{PM} = \underline{BM}$ or

$\underline{BM} + \underline{MP} = \underline{BP}$

but $\underline{BP} > \underline{BM}$, so

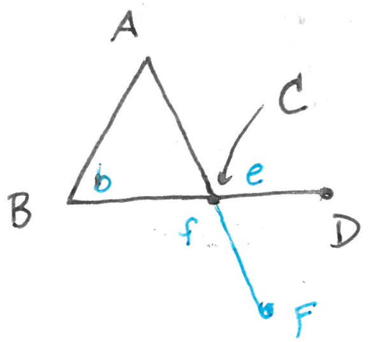
$\underline{BM} + \underline{MP} = \underline{BP}$.

substituting, we get

$\underline{BM} + \underline{MP} = 2\,\underline{BM}$

so $\underline{MP} = \underline{BM}$

Thm 40 b



Given $\triangle ABC$, with $\overline{BC}$ extended to $\overline{BD}$

Extend $\overline{AC}$ to $\overline{AF}$

Name $m\angle ABC = b$

$m\angle ACD = e$

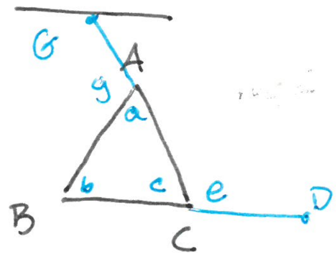
$m\angle BCF = f$

By 40a, $b < f$

By 39, $f = e$

By substitution $b < e$

Thm 41



Given $\triangle ABC$, name interior angle measures:

$m\angle BAC = a$ $\quad m\angle ABC = b$ $\quad m\angle ACB = c$

Extend $\overline{BC}$ to D.

Name exterior angle measure $m\angle ACD = e$

By thm 40 $\quad a < e^{(1)}, b < e^{(2)}$.

By thm 25 $\quad c + e = 180°$ (3)

By substitution $\quad 180° = c + e > c + a \quad (1, 3)$

$180° = c + e > c + b \quad (2, 3)$

Extend $\overline{CA}$ to $\overline{CG}$

name exterior angle measure $m\angle BAG = g$

By thm 40 $\quad g > b$

by thm 25 $\quad a + g = 180°$

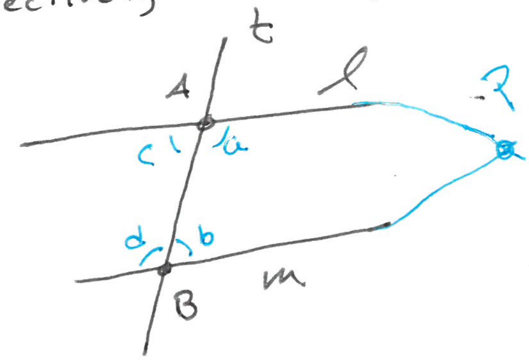
By substitution $180° = a + g > a + b$

So $a+b$, $a+c$, $b+c$ are all $< 180°$

Thm 44: Given that line t intersects lines l and m in points A and B respectively.

name the interior angle measures on one side of t a and b (at A and B respectively)

and name the interior angle measures on the other side c and d respectively



Suppose l and m are not parallel.

Then l and m intersect.

Name $P = l \cap m$.

Then $\triangle ABP$ is a triangle, and

either a and b are interior angle measures of $\triangle ABP$

or c and d are interior angle measures of $\triangle ABP$.

We are given $a+b=180°$ or $c+d=180°$

by Thm 43, both $a+b=180°$ and $c+d=180°$

which contradicts Thm 41

so l and m are parallel