Sets that are defined by distance formulas are generally preserved under isometries. For example:

Ellipses: An ellipse can be defined by its foci and a distance. The distance must be longer than the focal distance in order for the set to be non-empty and not a segment:

$$E(A,B,r) = \{X \mid d(A,X) + d(X,B) = r\}$$

Single branch of a hyperbola: A branch of a hyperbola can be defined by its foci and a distance greater than zero. Note that the order of the foci matters: switching the order of the foci will specify the other branch of the hyperbola:

$$H(A,B,r) = \{X \mid d(A,X) - d(B,X) = r\}$$

Ratio set: This set can be proven to always be a line or circle. It is defined as the locus of points that have a set ratio between their distance to one point or the other. Ellipses and Hyperbolas are classically named sets. My term “ratio set” is an informal, not a commonly named set. In this set definition, A and B are points, and r is a positive number.

$$R(A,B,r) = \{X \mid d(A,X) = r \cdot d(B,X)\}$$

Some conjectures that could be proven are:

> The isometric image of an ellipse is an ellipse.
>
> The isometric image of a branch of a hyperbola
>
> The isometric image of a ratio set, is a ratio set with the same ratio.

These conjectures could be more specific. For example:

> The isometric image of an ellipse is an ellipse with the same focal length and total distance
>
> The isometric image of an ellipse is an ellipse with the same focal length and eccentricity

In all cases, to prove the theorem, you will be naming points and distances so you can prove something using specific notation: For example:

> For an isometry, f, the isometric image of E(A,B,r) is E(f(A), f(B),r)