SAS proof outline:

Name the triangles and congruent sides and angle as givens. For example:

- Given triangles ΔABC and ΔDEF such that $\overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$ and $\overline{BC} \cong \overline{EF}$

Then use the application theorem, which matches one point exactly, and puts another point on a ray starting from the image of the first point. Make sure that those first two points that you map by the application theorem are endpoints of one given congruent sides.

So, these pairs work:				These pairs don't	
$A \to D$ $B \to \overrightarrow{DE}$	$B \to E$ $A \to \overrightarrow{ED}$	$C \to F$ $B \to \overrightarrow{FE}$	$B \to E$ $C \to \overrightarrow{EF}$	$A \to D$ $C \to \overrightarrow{DF}$	$C \to F$ $A \to \overrightarrow{FD}$

- By the application theorem, there is an isometry *f* such that $f(A) = D$, $f(B) \in \overrightarrow{DE}$ and $f(C)$ on the same side of $\overleftrightarrow{DE}$ as *F*.

Use the givens, the isometry property, and the point on the ray to use Theorem 2 to prove that the second point maps exactly where you want it to, so you have two points matched exactly, and one of them is B.

- Use theorem 2 to prove that $f(B) = E$

Now you have one side of the angle mapped exactly as you want it, so then you use theorem 1 to match the other side of the angle as a ray. You'll be proving that the third point of the triangle maps to a point on the ray of the angle in the other triangle. You need to use the given congruent angles, the isometry property applied to the angle, and the property from the application theorem about which side of a line the third point maps to.

- Use theorem 1 to prove that that $f(C) \in \overrightarrow{EF}$

Finally, prove that the third point lands exactly where you want it to, because it is at the right distance from the angle vertex. You can then conclude that the triangles are congruent.

- Use theorem 2 to prove that that $f(C) = F$
- Conclude that $\Delta ABC \cong \Delta DEF$

ASA proof outline:

Name the triangles and congruent side and angles as givens. For example:

- Given triangles ΔABC and ΔDEF such that $\overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$ and $\angle BAC \cong \angle EDF$

Then use the application theorem, which matches one point exactly, and puts another point on a ray starting from the image of the first point. Make sure that those first two points that you map by the application theorem are endpoints of the given congruent side.

Only these choices work:	
$A \to D$ $B \to \overrightarrow{DE}$	$B \to E$ $A \to \overrightarrow{ED}$

- By the application theorem, there is an isometry *f* such that $f(A) = D$, $f(B) \in \overrightarrow{DE}$ and $f(C)$ on the same side of $\overleftrightarrow{DE}$ as *F.*

Use the givens, the isometry property, and the point on the ray to use Theorem 2 to prove that the second point maps exactly where you want it to, so you have two points matched exactly, and one of them is B.

- Use theorem 2 to prove that $f(B) = E$

Now you have the shared side of the two angles that you have congruences for mapped exactly as you want it, so then you use theorem 1 to match the other sides of each angle as a ray. You'll be proving that the third point of the triangle maps to a point on the ray from one angle, and then that the point it is mapped to also lies on the ray from the other angle.

You need to use the given congruent angles, the isometry property applied to the angle, and the property from the application theorem about which side of a line the third point maps to.

- Use theorem 1 to prove that that $f(C) \in \overrightarrow{EF}$

- Then use theorem 1 again to prove that that $f(C) \in \overrightarrow{DF}$

Use theorem 3 to say that the intersection of the two rays $\overrightarrow{DF}$ and $\overrightarrow{EF}$ can contain at most one point, and since they contain both the image of the third point of the first triangle, and the third point of the second triangle, those points must be the same:

- Then use theorem 3 to say that because $f(C), F \in \overrightarrow{EF} \cap \overrightarrow{DF}$ then $f(C) = F$

- Conclude that $\Delta ABC \cong \Delta DEF$

Application proof outline:

State the given triangles. For example:

- ♦ Given triangles ΔABC and ΔDEF

Describe how to intersect circles to get a point at an equal distance from the first pair of points, and name it. Define and name a rotation around that point that will map one point to the other

- ♦ Let *P* be a point at the intersection of circles with centers A and D respectively, and radius $d(A,D)$, and let f be the rotation around point P by angle $\angle APD$

Prove that the rotation maps the first point to the correct place:

- ♦ Use theorem 1 to prove that $\overrightarrow{Pf(A)} = \overrightarrow{PD}$
- ♦ Use theorem 2 to prove that $f(A) = D$

Define a rotation that fixes the first point, and maps the image of the second point onto the desired ray:

- ♦ Let g be the rotation around point D by angle $\angle f(B)DE$

Prove that the composition of the two rotations keeps the image of the first point at the desired image, and maps the second point to the desired ray:

- ♦ Use theorem 1 to prove that $\overrightarrow{Pg(f(B))} = \overrightarrow{DE}$
- ♦ Use the fixed point property of the rotation to prove that $g(f(A)) = D$
- ♦ Use theorem 8 to prove that $g \circ f$ is an isometry

At this point, you have an isometry that satisfies the first two conditions of the application theorem conclusion: the image of A and the image of B. There are two cases depending on where the image of C is under the isometry $g \circ f$. You will need to state each case and prove that the theorem holds in both cases.

- ♦ State case 1: if $g(f(C))$ is on the same side of $\overleftrightarrow{DE}$ as F, then $g \circ f$ is an isometry that satisfies the conditions of the theorem
- ♦ State case 2: if $g(f(C))$ is on the opposite side of $\overleftrightarrow{DE}$ from F, then define h to be the reflection across the line $\overleftrightarrow{DE}$
- ♦ Use the fixed point property of the reflection to verify that $h(g(f(A))) = D$ and $h(g(f(B))) \in \overrightarrow{DE}$
- ♦ Use the side reversal property of the reflection to prove that $h(g(f(C)))$ is on the same side of $\overleftrightarrow{DE}$ as F
- ♦ Use theorem 8 to prove that $h \circ g \circ f$ is an isometry
- ♦ Note that $h \circ g \circ f$ satisfies the conditions of the application theorem.
- ♦ Conclude that the theorem is satisfied in both cases, and is therefore proven.