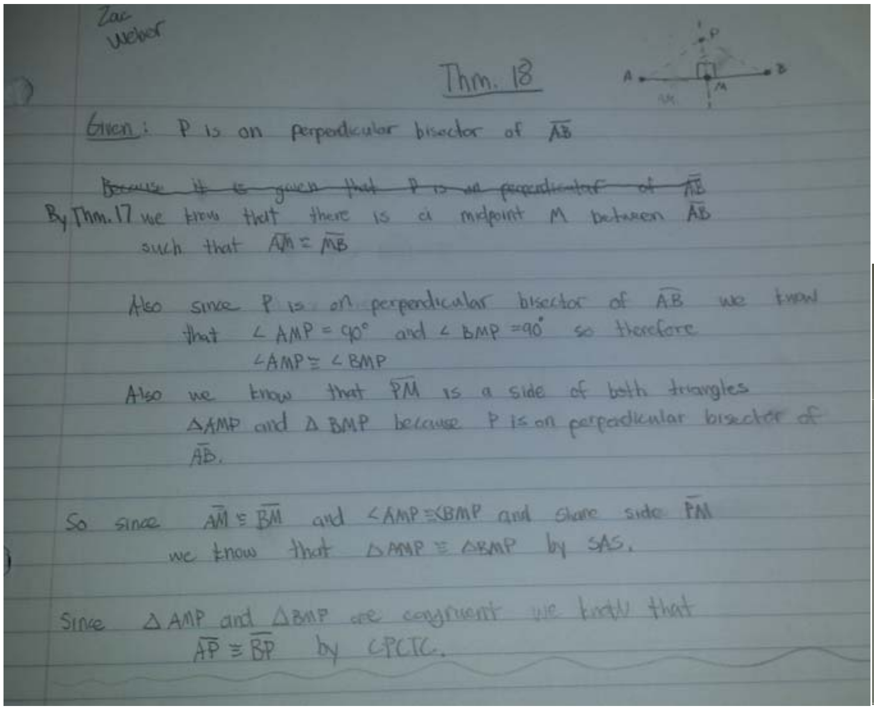
Theorem 18



Zac Weber

Thm. 18

Given: P is on perpendicular bisector of $\overline{AB}$

~~Because it is given that P is on perpendicular of $\overline{AB}$~~

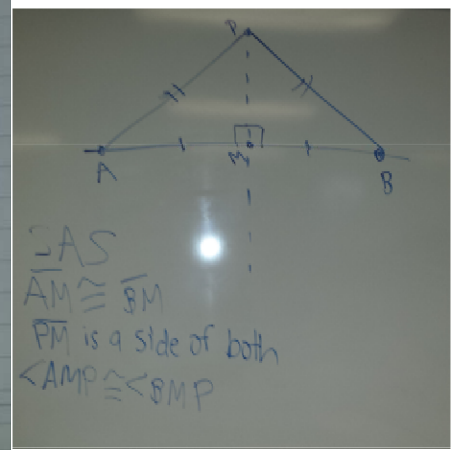
By Thm. 17 we know that there is a midpoint M between $\overline{AB}$ such that $\overline{AM} \cong \overline{MB}$

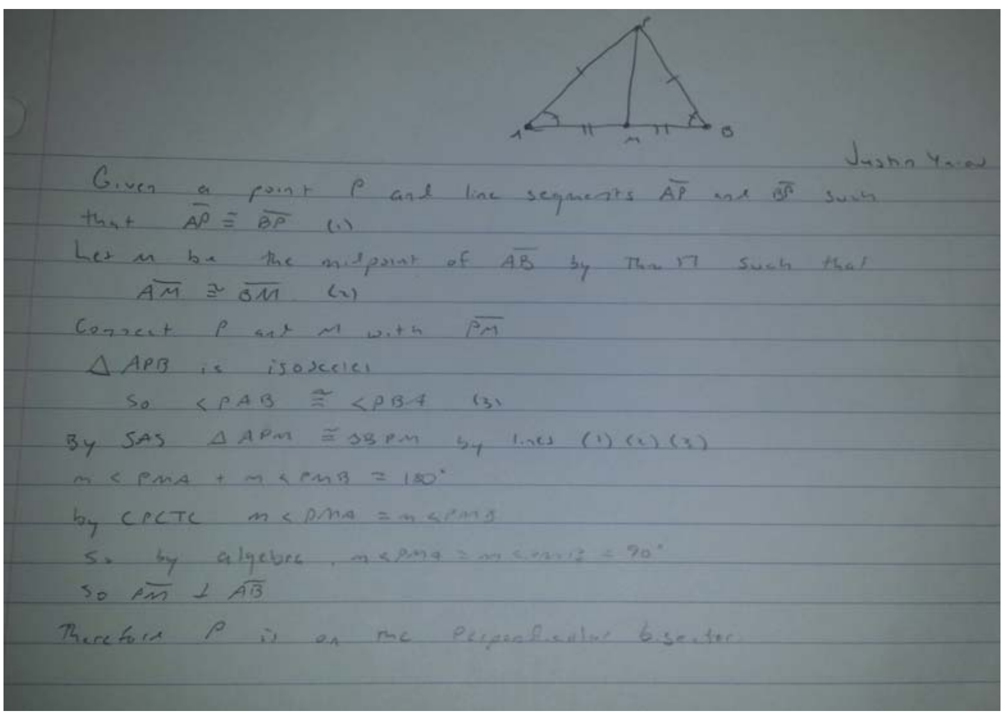
Also since P is on perpendicular bisector of $\overline{AB}$ we know that $\angle AMP = 90°$ and $\angle BMP = 90°$ so therefore $\angle AMP \cong \angle BMP$

Also we know that $\overline{PM}$ is a side of both triangles $\triangle AMP$ and $\triangle BMP$ because P is on perpendicular bisector of $\overline{AB}$.

So since $\overline{AM} \cong \overline{BM}$ and $\angle AMP \cong \angle BMP$ and share side $\overline{PM}$ we know that $\triangle AMP \cong \triangle BMP$ by SAS.

Since $\triangle AMP$ and $\triangle BMP$ are congruent we know that $\overline{AP} \cong \overline{BP}$ by CPCTC.





Justin Y.

Given a point P and line segments $\overline{AP}$ and $\overline{BP}$ such that $\overline{AP} \cong \overline{BP}$ (1)

Let M be the midpoint of $\overline{AB}$ by Thm 17 such that $\overline{AM} \cong \overline{BM}$ (2)

Connect P and M with $\overline{PM}$

$\triangle APB$ is isosceles

So $\angle PAB \cong \angle PBA$ (3)

By SAS $\triangle APM \cong \triangle BPM$ by lines (1) (2) (3)

$m\angle PMA + m\angle PMB = 180°$

by CPCTC $m\angle PMA = m\angle PMB$

So by algebra, $m\angle PMA = m\angle PMB = 90°$

So $\overline{PM} \perp \overline{AB}$

Therefore P is on the perpendicular bisector

Thm 17 parts 2 and 3

Thm 17: Part B

(1) Suppose there are 2 unique midpoints on line $\overline{AB}$ called m_1 + m_2.

(2) ~~So, $d(A,m_1)+d(m_1,m_2)+d(m_2,B) = d(A,B)$~~

(3) $d(A,m_1)+d(m_1,B)=d(A,B)$ By defn. of midpoint

(4) $d(A,m_2)+d(m_2,B)=d(A,B)$

(5) $\overline{Am_1} \cong \overline{Am_2} \cong \overline{m_2B} \cong \overline{m_1B}$ + lines 3+4
Algebra -Also by definition of midpoint.

(6) So there is a ray $\overrightarrow{Am_2}$ such that $m_1 \in \overrightarrow{Am_2}$ + by line (5)

(7) $\overline{Am_1} \cong \overline{Am_2}$

(8) By Thm 2, $m_1 = m_2$

(9) This contradicts line 1

So, there can't be more than one midpoint.

Therefore, there can be at most one midpoint. □

$$d(A,M_1)+d(B,M_1)=d(A,B)$$
$$d(A,M_2)+d(B,M_2)=d(A,B)$$

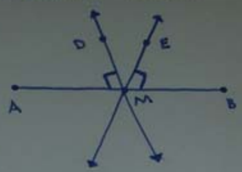
$$d(A,M_1)+d(B,M_1)=d(A,M_2)+d(B,M_2) \qquad \overline{AM_1}\cong\overline{BM_1}$$
$$\overline{AM_2}\cong\overline{BM_2}$$
$$d(B,m_1)+d(B,m_1)=d(B,m_2)+d(B,m_2)$$
$$\frac{2\,d(B,m_1)}{2}=\frac{2\,d(B,m_2)}{2}$$
$$d(B,m_1)=d(B,m_2)$$

Theorem 17 - Part 3

For any segment, there can't be more than one perpendicular bisector

Proof

Let M be the midpoint of $\overline{AB}$



Suppose there is more than one perpendicular bisector of $\overline{AB}$

Name two of these perpendicular bisectors $\overleftrightarrow{DM}$ and $\overleftrightarrow{EM}$ (distinct)

Then $m\angle DMA = 90°$ and $m\angle EMB = 90°$
(by the definition of perpendicular bisector)

Then $m\angle AMD + m\angle DME + m\angle EMB = 180°$
(by the angle axiom)

Using substitution, $90° + m\angle DME + 90° = 180°$

Thus, $m\angle DME = 0°$

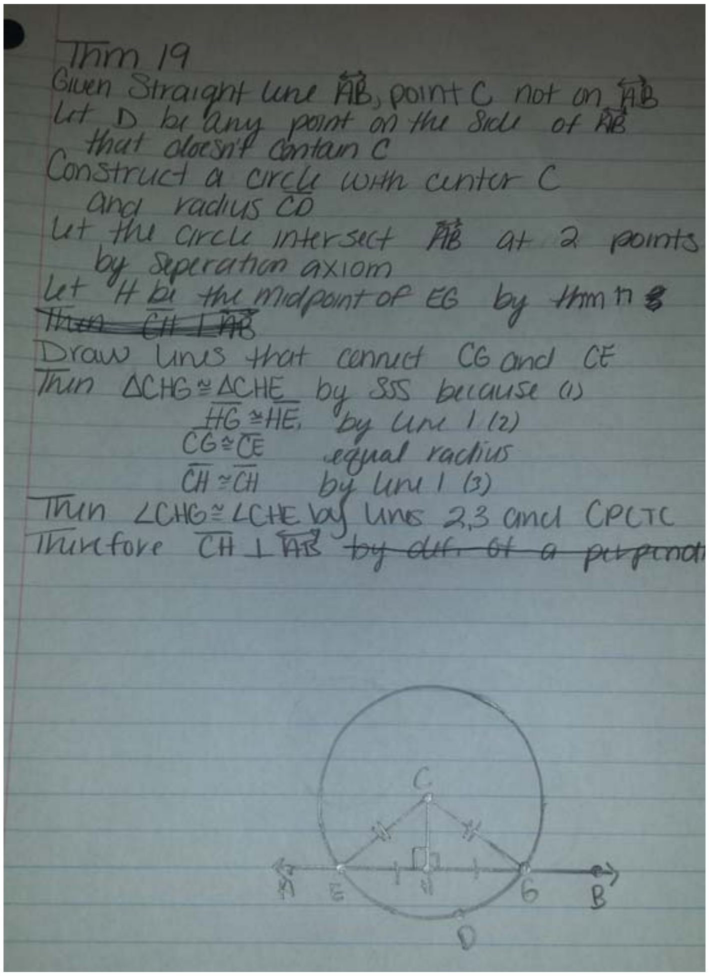
By the angle axiom, $\overleftrightarrow{DM} = \overleftrightarrow{EM}$ which contradicts there being more than one perpendicular bisector

Therefore, there is only one perpendicular bisector of a segment

$\blacksquare$

Thm 19

Part 1: given a line and a point not on the line, there exists a line perpendicular to the given line that includes the given point.



Thm 19

Given Straight line $\overleftrightarrow{AB}$, point C not on $\overleftrightarrow{AB}$

Let D be any point on the side of $\overleftrightarrow{AB}$ that doesn't contain C

Construct a circle with center C and radius $\overline{CD}$

Let the circle intersect $\overleftrightarrow{AB}$ at 2 points by Seperation axiom

Let H be the midpoint of EG by thm 11 ~~8~~

~~Then $\overline{CH} \perp \overleftrightarrow{AB}$~~

Draw lines that connect CG and CE

Then $\triangle CHG \cong \triangle CHE$ by SSS because (1)

$\overline{HG} \cong \overline{HE}$, by line 1 (2)

$\overline{CG} \cong \overline{CE}$ equal radius

$\overline{CH} \cong \overline{CH}$ by line 1 (3)

Then $\angle CHG \cong \angle CHE$ by lines 2,3 and CPCTC

Therefore $\overline{CH} \perp \overleftrightarrow{AB}$ ~~by def. of a perpendi~~

Part 2: Given a line and a point not on the line, there exists at most one line (ie. there cannot be 2 lines) that is perpendicular to the given line that includes the given point

Part 3: Given a line and a point on the line, there exists a line perpendicular to the given line that includes the given point.

Part 4: Given a line and a point on the line, there exists at most one line (ie. there cannot be 2 lines) that is perpendicular to the given line that includes the given point.