Application theorem

(1) Given triangles ΔABC and ΔDEF

(2) Construct circles C_1 and C_2 with centers A and D respectively, and with radius $d(A,D)$, and let P be a point in the intersection of C_1 and C_2

line 15

(3) Because C_1 and C_2 have the same radius, then $\overline{PA} \cong \overline{PD}$

(4) Let f be the rotation around point P by angle $\angle APD$ (so that $f(A)$ is on the same side of $\overleftrightarrow{AP}$ as D).

(5) Because f is a rotation, $\angle APf(A) \cong \angle APD$ and $f(P) = P$

(6) $\overrightarrow{Pf(A)} = \overrightarrow{PD}$ by theorem 1 and lines ___5__ and ___4____

(7) Because f is an isometry, $\overline{PA} \cong \overline{f(P)f(A)}$

(8) By lines __7___, ___5____ and __3____, $\overline{PD} \cong \overline{Pf(A)}$

(9) By theorem 2 and line ____6_____ and ____8____ $f(A) = D$

(10) Let g be the rotation around point D by angle $\angle f(B)DE$ (where $g(f(B))$ is on the same side of $\overleftrightarrow{Df(B)}$ as E)

(11) Because g is a rotation, $\angle f(B)Dg(f(B)) \cong \angle f(B)DE$ and $g(D) = D$

(12) By theorem 1 and lines ____10___ and ___11______, $\overrightarrow{Dg(f(B))} = \overrightarrow{DE}$

(13) By lines __9___ and _____11____, $g(f(A)) = D$

(14) By theorem 8 and lines ___4__ and ___10___, $g \circ f$ is an isometry

(16) If $g(f(C))$ lies on the same side of $\overleftrightarrow{DE}$ as F, then $g \circ f$ satisfies the conditions for the conclusion to be true by lines __14____, ____13___ and ___12_____.

(17) If $g(f(C))$ lies on the opposite side of $\overleftrightarrow{DE}$ from F, then let h be the reflection in the line $\overleftrightarrow{DE}$

(18) Since h is a reflection, $h(g(f(C)))$ lies on the opposite side of $\overleftrightarrow{DE}$ from $g(f(C))$

(19) By lines ____17___ and ____18____, $h(g(f(C)))$ lies on the same side of $\overleftrightarrow{DE}$ as F

(20) Since h fixes points on $\overleftrightarrow{DE}$, and by line __13___, $h(g(f(A))) = g(f(A)) = D$

(21) Since h fixes points on $\overleftrightarrow{DE}$, and by line __15___, $h(g(f(B))) = g(f(B))$

(22) So by lines __12__ and __21____, $h(g(f(B))) \in \overrightarrow{DE}$

(23) By theorem 8 and lines __17___ and __14____, $h \circ g \circ f$ is an isometry

(24) Then by lines ____23______, _____22____, ____20______ and ____19______, $h \circ g \circ f$ satisfies the conditions for the conclusion to be true.

Application theorem:

(1) Given triangles ΔABC and ΔDEF

(2) Construct circles C_1 and C_2 with centers A and D respectively, and with radius $d(A,D)$, and let P be a point in the intersection of C_1 and C_2

(3) Let f be the rotation around point P by angle $\angle APD$ (where $f(A)$ is on the same side of $\overrightarrow{AP}$ as D).

(4) Because f is a rotation, | then $\angle APf(A) \cong \angle APD$ and $f(P)=P$

(5) By theorem 1 and lines 3 and 4, | then $\overrightarrow{Pf(A)} = \overrightarrow{PD}$

(6) Because f is an isometry, | then $\overline{PA} \cong \overline{f(P)f(A)}$

(7) By lines 6 and 4, | then $\overline{PA} \cong \overline{Pf(A)}$

(8) By theorem 2 and lines 7 and 5 | then $f(A)=D$

(9) Let g be the rotation around D by angle $\angle f(B)DE$ (where $g(f(B))$ is on the same side of $\overrightarrow{Df(B)}$ as E)

(10) Because g is a rotation, | then $\angle f(B)Dg(f(B)) \cong \angle f(B)DE$ and $g(D)=D$

(11) by theorem 1 and lines 10 and 9, | then $\overrightarrow{Dg(f(B))} = \overrightarrow{DE}$

(12) By theorem 8 and lines 3 and 9, | then $g \circ f$ is an isometry

(13) By lines 8 and 10, | then $g(f(A)) = D$

(14) By lines 13 and 11, | then $g \circ f$ maps A to D and maps B to a point on $\overrightarrow{DE}$

(15) If $g(f(C))$ lies on the same side of $\overleftrightarrow{DE}$ as F then by lines 14, 12 and [illegible], | then $g \circ f$ satisfies the conditions for the conclusion to be true

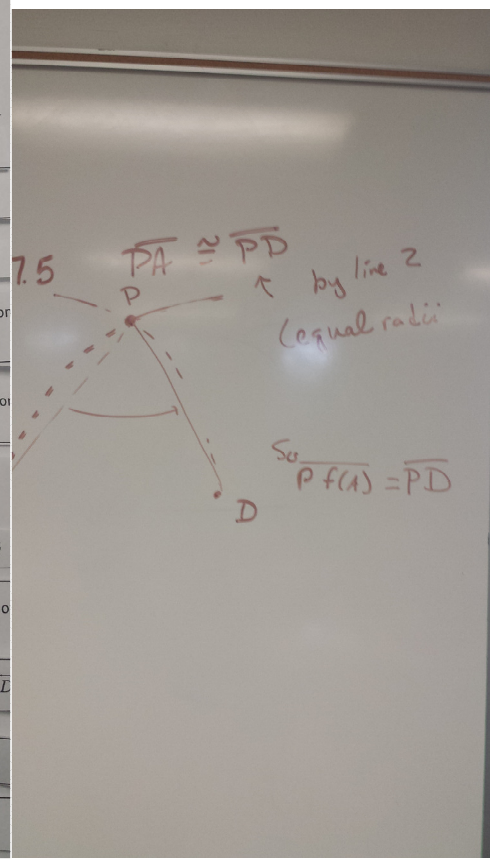
(16) If $g(f(C))$ lies on the opposite side of $\overleftrightarrow{DE}$ from F, then let h be the reflection across the line $\overleftrightarrow{DE}$

(17) Since h is a reflection, | then $h(g(f(C)))$ lies on the opposite side of from $g(f(C))$

(18) Since h fixes points on $\overleftrightarrow{DE}$, and by line 16, | then $h(g(f(C)))$ lies on the same side of $\overleftrightarrow{D}$

(19) Since h fixes points on $\overleftrightarrow{DE}$, and by line 14, | then $h(g(f(B))) = g(f(B))$

(20) By lines 19 and 14, | then $h(g(f(B))) \in \overrightarrow{DE}$



By lines 13 and 16, | then $h(g(f(A))) = g(f(A)) = D$

By theorem 8 and lines 16 and 12, | then $h \circ g \circ f$ is an isometry

Then by lines 22, 21, 20 and 18, $h \circ g \circ f$ satisfies the conditions for the conclusion to be true.