25. Prove that the composition of two 1-1 functions is 1-1.

Proof: Given functions  and  that are both 1-1 functions.

This means:

|  |  |
| --- | --- |
| If  for any  then   | If  for any  then   |

There is a function  (because the codomain of *f* is the domain of *g*.)

Suppose  for some 

Then  and  where  and 

Since *g* is 1-1, we know that 

So 

Since *f* is 1-1, we know that 

This if  for some , then .

This proves that  is 1-1.

26. Prove that the composition of two onto functions is onto.

Proof: Given functions  and  that are both onto functions.

This means

|  |  |
| --- | --- |
| If *u* is any element in *Y*, then somewhere in *X* there is an element *a* that maps to *u* (so  ) | If *t* is any element in *Z*, then somewhere in *Y* there is an element *v* that maps to *t* (so  ) |
| There is a function   | (because the codomain of *f* is the domain of *g*.) |
| Let   | (This means: pick any element in *Z* and name it *r*). |
| Because *g* is onto, there exists  such that   | This means: somewhere in *Y* there is an element that maps to *r*; let’s name it *w*. |
| Now because  and *f* is onto, there exists  such that   | This means: somewhere in *X* there is an element that maps to *w*, let’s name it *b*. |
| So,   |  |
| We have shown that given any element , there exists and element  such that , and hence  is onto. |  |