Math 411

Game of Mathematics

Invented / Popularized by Euclid

(~300 BC)

Euclidean Geometry

To start the game, all of the players (mathematicians) agree on a small number of rules (at least 1)

Axioms (postulates)

The following moves can earn you points in the game:

- Propose a new rule (make a conjecture)—the rule must be accepted by the other players as true or interesting or both to earn points
- Prove that a new rule should be accepted

A new rule can be accepted either as an Axiom (Postulate) or as a Theorem (Proposition)

- To prove a **Theorem** should be accepted, you must show why, using logical arguments, the theorem has to be true if all of the previously accepted axioms (and theorems) are true.

A new rule can be accepted either as an Axiom (Postulate) or as a Theorem (Proposition)

- To prove an **Axiom** should be accepted you have to prove that:
  - The Axiom is consistent with the other axioms (there are no contradictions—all of them can be true simultaneously)
  - The Axiom is independent of the other axioms (it can't be proven as theorem)
  - The Axiom is useful (there are situations we want to know about where the Axiom is true)

You can also earn points by

- Explaining something better than anyone else or
- Disproving a previously accepted claim (either a postulate or a previously accepted proof)

History:Axioms

Euclid's first 5 Axioms/Postulates were:

Postulate 1.
To draw a straight line from any point to any point.

Postulate 2.
To produce a finite straight line continuously in a straight line.

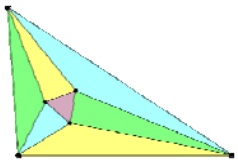
Postulate 3.
To describe a circle with any center and radius.

Postulate 4.
That all right angles equal one another.

Postulate 5.
That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

History: Making a Conjecture

Morley's Theorem: The adjacent trisectors of the angles of a triangle are concurrent by pairs at the vertices of an equilateral triangle (proposed and proved the theorem)



History: Making a Conjecture

Poincaré conjecture: Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.

Originally conjectured by Henri Poincaré (1854-1912)
Proved by Grigori Perelman in 2003

History: Making a Conjecture

Euclid's fifth postulate can be proved using the first four postulates (conjectured by: Ptolemy, al-Gauhary, al-Haytham, Omar Khayyam, A.M.Legendre)

Finally disproved by: Gauss, J. Bolyai and Lobachevsky)

History: Making a Conjecture

There are an infinite number of pairs of twin primes (open problem—not proven)