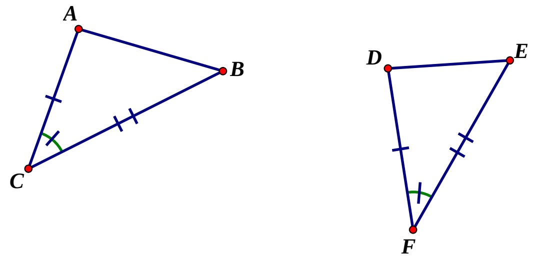
Quiz practice

1. Given triangles below, define an isometry that maps ΔABC to ΔDEF, matching congruent parts in useful ways as much as possible:



The smartest choice is to first map C to F, because that's the angle given. However, if you first matched one of the other vertices, it's still OK if the next thing you do is map C to a correct side. Because the diagrams are given, you know that a reflection is not needed, so this solution does not have two cases. Here is one solution:

There is a translation, *T*, such that *T(C)=F*

There is a rotation, ρ, such that $\rho(F) = F$ and $\rho(T(A)) \in \overrightarrow{FD}$

Note: you must specify both the fixed point and the ray the point is rotated onto.

By theorem 8 $f = \rho \circ T$ is an isometry.

2. Given triangles ΔABC and ΔDEF with congruent angles and sides: $\overline{AC} \cong \overline{DF}$ and $\angle ACB \cong \angle DFE$, define an isometry that maps ΔDEF to ΔABC, matching congruent parts in useful ways as much as possible.

Because you are only given one congruent side and one congruent angle, you must choose to match one of the endpoints of the segment first, and the other endpoint segment.

Because you are not given a diagram, you will need to do 2 cases to match the side of the third vertex.

Note that in #1 you are mapping ΔABC to ΔDEF *and in this problem the mapping goes in the other direction. Here is one solution:*

Let *T* be a translation such that $T(F) = C$

Let ρ be a rotation such that $\rho(C) = C$ and $\rho(T(D)) \in \overrightarrow{CA}$

Case 1: $\rho(T(E))$ is on the same side of $\overleftrightarrow{CA}$ as *B* Let $f = \rho \circ T$. By theorem 8, f is an isometry.	Case 2: $\rho(T(E))$ is on the opposite side of $\overleftrightarrow{CA}$ from *B* Let *R* be a reflection that fixes $\overleftrightarrow{CA}$.* Let $f = R \circ \rho \circ T$. By theorem 8, f is an isometry.

Note that I will accept either the wording "Let __ be a [type of isometry]" or the wording "There exists a [type of isometry], __" in defining the isometries.

**Alternate wording: "Let *R* be a reflection that is the identity on $\overleftrightarrow{CA}$". You may optionally add the phrase "that maps $\rho(T(E))$ to the same side of $\overleftrightarrow{CA}$ as *B*

3. Given triangles ΔABC and ΔDEF with congruent angles and sides: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\angle ABC \cong \angle DEF$, and an isometry f such that $f(A) = D$, $f(B) \in \overrightarrow{DE}$ and $f(C)$ on the same side of $\overleftrightarrow{DE}$ as *F*.
Prove that $f(C) = F$ and prove that $\Delta ABC \cong \Delta DEF$

Note that you do not need to redefine the isometry—I already did that for you. Just do the proof part. Start with the vertex you know, and work towards the ones you don't know yet.

Proof:

$f(A) = D$ (given)

Note: it's OK to rename f(B) and f(C) to B' and C'

$\overline{AB} \cong \overline{f(A)f(B)} = \overline{Df(B)}$ (*f* is an isometry)
$\overline{AB} \cong \overline{DE}$ (given)
so $\overline{DE} \cong \overline{Df(B)}$
$f(B) \in \overrightarrow{DE}$ (given)
So by theorem 2, $f(B) = E$

$\angle ABC \cong \angle DEF$ (given)
$\angle ABC \cong \angle f(A)f(B)f(C) = \angle DEf(C)$ (*f* is an isometry)
so $\angle DEF \cong \angle DEf(C)$
$f(C)$ is on the same side of $\overleftrightarrow{DE}$ as *F*. (given)
So by theorem 3 $\overrightarrow{EF} = \overrightarrow{Ef(C)}$ which means $f(C) \in \overrightarrow{EF}$

$\overline{BC} \cong \overline{EF}$ (given)
$\overline{BC} \cong \overline{f(B)f(C)} = \overline{Ef(C)}$ (*f* is an isometry)
so $\overline{EF} \cong \overline{Ef(C)}$
$f(C) \in \overrightarrow{EF}$ (proved above)
So by theorem 2, $f(C) = F$

By theorem 9 $\Delta ABC \cong \Delta f(A)f(B)f(C)$
and by substitution $\Delta f(A)f(B)f(C) = \Delta DEF$
So, $\Delta ABC \cong \Delta DEF$
QED