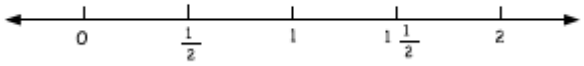
Estimate the sum: $\frac{12}{13}+\frac{7}{8}$. The choices are: 1, 2, 19, or 21.	Explain that only 24% of eighth graders on a national test could do this correctly. Ask: What is a reasonable estimate? Why did so many students choose 19 or 21? Why is addition and subtraction of fractions so hard for so many students?
Present this task and ask students to consider if this solution makes sense. $\frac{1}{3}+\frac{8}{9}=\frac{9}{12}$ Do you agree? Where would the answer go on this number line? 	After students share their ideas guide the discussion with these questions: • Let's use our fraction order ideas to judge the reasonableness of this answer: Is $\frac{8}{9}$ > or < $\frac{1}{2}$? [Using ½ as a benchmark] • What would you add to $\frac{8}{9}$ to equal one whole? [Residual idea] • Is $\frac{1}{3}$ > or < $\frac{1}{9}$? •Is the sum > or < 1? [Comparing fractions with same numerator] • Is $\frac{9}{12}$ > or < 1?
Present this story to the students: William ate 1/4of a pizza for dinner. The next morning he ate a piece that equaled 1/8 of the pizza. How much of a pizza did he eat? Explain that you don't want the exact answer, but just an estimate. Ask students to imagine 1/4 of a pizza and 1/8 of a pizza. Did William eat more or less than 1/2 of a pizza? Have students to explain their responses by referring to their mental images for 1/4 and 1/8 . Explain to students that some people would say that 1/8 + 1/4 is 2/12 . Ask: Does that make sense? If you ate 1/4 and then 1/8 of a pizza would that be the same as 2/12 ? Show with circles 1/4 , 1/8 , and 2/12 of the black circle.	Students' explanation of estimation may sound like this: (a) He ate less than 1/2 . You need two-fourths to be 1/2 , and 1/8 is less than 1/4 . (b) Students may give a fraction circle manipulative explanation

Alice noticed that there was 3/4 of a pizza left after the party. She ate a slice of pizza that was the size of 1/8 of a whole pizza. About much pizza was left after Alice ate a slice? Ask for estimates. Will there be more or less than 1/2 pizza left? Try picturing the 3/4 pizza in your mind. Does this help in your estimate? Explain your thinking.	
Joe & Renata each receive the same allowance. Joe spent 2/3 of his allowance on records. Renata spent 1/6 of her allowance repairing her bicycle. How much more did Joe spend than Renata?	
Marge ran 3 4 mile and stopped to catch her breath. She then ran another 1 8 mile. Did Marge run more or less than one mile? Did Marge run 4 12 mile?	Possible student response: • It has to be greater than 1/2 but less than 1. To make one whole you'd need to add 1/4 more. But 1/8 < 1/4 so it's not 1 mile. • 4/12 doesn't make sense because 4/12 < 1/2 .

What reasoning are these students using?

a. $\frac{11}{12}-\frac{1}{3}=\frac{10}{12}$ This doesn't make sense. $\frac{11}{12}$ is close to 1; so 1 – $\frac{1}{2}$ = $\frac{1}{2}$. $\frac{10}{12}$ is closer to 1 than $\frac{1}{2}$.

b. $\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$. Yes, I do think this is reasonable. $\frac{1}{3}$ is close to 1 and $\frac{1}{4}$ is close to $\frac{1}{3}$. If you add them you are close to 1. Also $\frac{11}{12}$ is close to 1.

c. $\frac{1}{5}+\frac{2}{3}=\frac{3}{5}$. Doesn't make sense. $\frac{2}{3}$ is bigger than $\frac{2}{5}$. It would be closer to $\frac{4}{5}$.

d. $\frac{2}{3}-\frac{1}{4}=\frac{1}{12}$. Doesn't make sense. $\frac{1}{4}$ is smaller than $\frac{1}{3}$. $\frac{1}{12}$ is smaller than $\frac{1}{4}$ More than $\frac{1}{3}$ is left.

e. $\frac{8}{15}-\frac{1}{3}=\frac{7}{12}$. $\frac{8}{15}$ is practically the same thing as $\frac{7}{12}$; both a little larger than $\frac{1}{2}$. And you're taking away a little bit less than $\frac{1}{2}$. So, no, it doesn't make sense.

<u>What would improve these students' reasoning? (What did they leave out or overlook)</u>

a. $\frac{9}{10}-\frac{2}{100}=\frac{7}{10}$. This makes sense. $\frac{9}{10}$ is almost 1. $\frac{2}{100}$ is almost 0. $\frac{7}{10}$ is almost 1. 1-0=1

b. $\frac{9}{10}-\frac{2}{100}=\frac{7}{10}$. This looks right because $\frac{9}{10}$ is close to one and $\frac{2}{100}$ is very close to 0. So the answer would be between 1/2 and one.

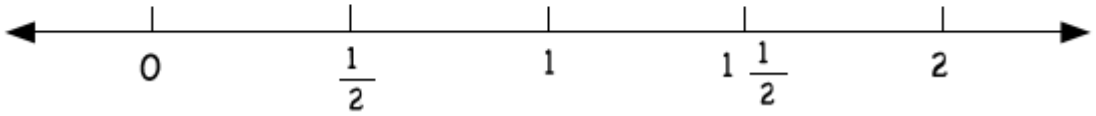
c. $\frac{1}{4}-\frac{2}{100}=\frac{1}{3}$. Doesn't make sense because $\frac{1}{4}$ is less than $\frac{1}{2}$ and $\frac{2}{100}$ is almost 0.

d. $\frac{1}{4}-\frac{2}{100}=\frac{1}{3}$. Wrong! $\frac{2}{100}$ is nothing. $\frac{1}{4}$ - nothing does not equal $\frac{1}{3}$.

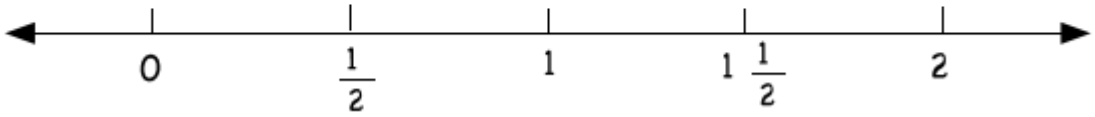
e. $\frac{8}{15}-\frac{1}{3}=\frac{7}{12}$. This does not make sense. $\frac{8}{15}$ is a little bit more than $\frac{1}{2}$. And $\frac{1}{3}$ is a little less than $\frac{1}{2}$.

Estimate each sum or difference. Explain your reasoning in words.

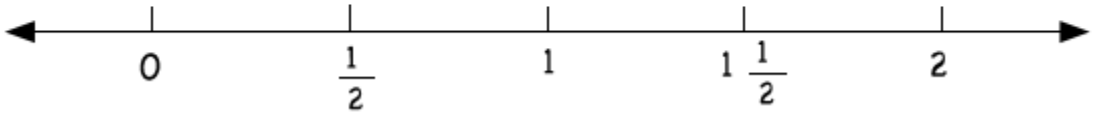
1. $\frac{1}{4}+\frac{1}{3}$



2. $\frac{5}{6}+\frac{11}{12}$



3. $1\frac{9}{10}-\frac{3}{4}$



4. $1\frac{7}{14}-\frac{9}{10}$