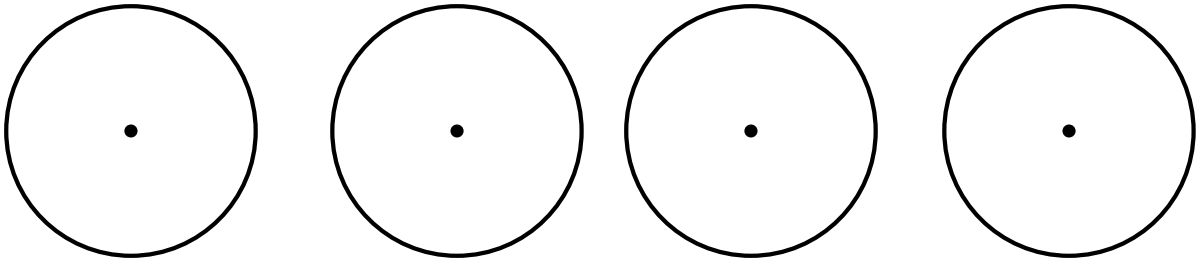
Fractions, Ratios and the Number system:

Models: We're going to use three of the most common models or visualizations for fractions: geometric fractions of a circle and a rectangle, and number line fractions of a length. Each of these models has advantages and disadvantages for learning about fractions. In grade 2, students are introduced to fractions using geometric shapes (such as circles and rectangles) and in grade 3, students are introduced to fractions on a number line.

Most of the visualizations we draw for fractions, are built on an area model of a fraction: the relationship between the area of the part and the area of the whole represents the fraction.

In a **circle** model, the circle represents the whole. The circle is subdivided using radii (segments from the center of the circle to the perimeter), that cut the circle into sectors with equal area. These sectors represent the fractional amount when compared to a whole that is represented by the entire circle.

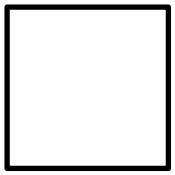
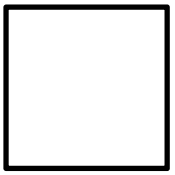
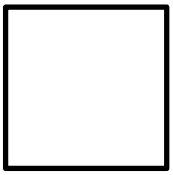
A circle model of fractions has the advantage that our brains are pretty good at recognizing and estimating angle sizes, so the visualization of the size of a third or a fourth or a fifth is something we're pretty good at remembering and estimating. Practice your estimation strategies on these circles by subdividing them:



thirds fourths fifths sixths

In a **rectangular** model, a rectangle or square (a square is just a special kind of rectangle) represents the whole. The whole is then subdivided using horizontal and vertical lines to make equal sized parts. Rectangles and squares can also be divided into halves or fourths using diagonal lines, but those aren't going to give us the most useful properties that rectangular models have, so we're going to just use subdividing lines that are parallel to the sides.

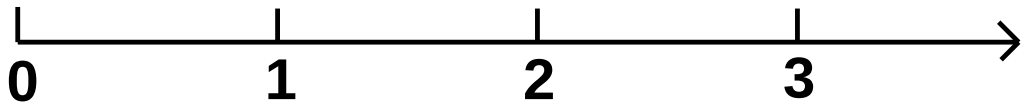
Rectangular models get really useful when you are making subdivisions in both directions at once. For instance, you could subdivide these squares square into sixths in two ways: either by making all of the subdivisions parallel to the same side, or by making some dividing lines vertical and others horizontal.

 Show sixths using only vertical lines	 Show sixths using some vertical and some horizontal lines	In the pictures of sixths you just drew, can you see thirds in the same picture? Can you see halves?	 How would you draw 20ths on a rectangular diagram?

The **number line** model of fractions shows a fraction as a length. The length of a whole unit is marked and labelled as 1 (it is a marked length not a separate shape like a circle or a square). Fraction bars and tape diagrams (which are also called bar diagrams) are variations on the number line model.

Number line fractions are particularly good for representing improper fractions, for comparing fractions to whole numbers, and for thinking of fractions as a kind of number rather than a kind of shape.

On this number line, show the fractions 2/3, and 5/2:



CCSS: In grade 2 children work with simple fractions of geometric shapes:

> CCSS.Math.Content.2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths…

In grade 3, the number line model for fractions is introduced, and is a topic of particular focus:

> CCSS.Math.Content.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

Building on Unit Fractions: Understanding fractions as numbers (grades 3-5)

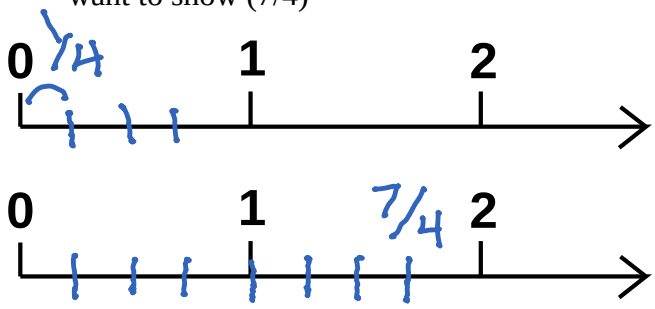
The most important idea for basic fraction knowledge in the common core standards is how to present fractions in terms of unit fractions, as described below:

A **unit fraction** is a fraction whose numerator is 1. To show the unit fraction 1/b in any of our models, we start with a representation of 1 whole, and divide it into b equal parts. Each of those parts represents the fraction 1/b. This is always the way that these fractions are defined and presented, so only the name *unit fraction* should be new to you.

Fractions with a numerator greater than 1 are represented and explained as a sum of unit fractions, so 2/3 is 2 units of size 1/3, and 9/4 is 9 units of size ¼. This is slightly different from the most common way of presenting such fractions. The most common way of explaining 2/3 is to say that it is two out of 3 equal parts of a whole. That's a fine explanation for fractions that are less than 1 (proper fractions), but it leads to misunderstandings when children encounter fractions that are greater than 1 (improper fractions). This is the most fundamental change suggested for teaching and understanding fractions in the Common Core Standards compared to the way things have usually been done in the past, and it is a topic of emphasis in grade 3.

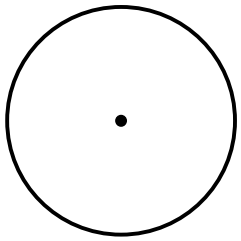
Practice: Using a number line and a circle model, show the steps (using unit fractions) to represent the fraction 7/4. Write a sentence explaining each step.

- The first step is to divide the whole unit into equal parts to show the size of the unit fraction (1/4)
- The second step is to draw out the right number of unit fractions to make the fraction you want to show (7/4)



Mark the unit length 1 on the number line, and partition it into 4 equal parts (1/4)

Draw 7 parts that are each 1/4 long and label the end 7/4

Unit fraction 	
Repeated units:	

CCSS: Understanding fractions as repeated unit fractions is an explicit standard at third grade, and is again reiterated at fourth grade:

> CCSS.Math.Content.3.NF.A.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
>
> CCSS.Math.Content.4.NF.B.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.