On Monday, we came up with nonsense word names for numbers. The first 4 number names were:

ounce dice trice quartz

Then we snapped together 5 cubes to make a long stick of cubes and brainstormed on what to call it. Someone suggested Kevin, because Kevin Garnett wears the number 5.

We then added some more loose (not snapped together) cubes and got

Kevin ounce Kevin dice Kevin trice Kevin quartz

and then of course we had to snap them together when we got the next cube, so we had dice Kevin.

Eventually we had 5 Kevins, and those had to be grouped together, and we got a new place value, which we voted to called Garnett

So we had a lot of numbers like

Garnett trice Kevin dice

and

Dice Garnet Kevin quartz.

We used a verson of normal numerals for these

On Monday, we came up with nonsense word names for numbers. The first 4 number names were:

ounce 1 dice 2 trice 3 quartz 4

Then we snapped together 5 cubes to make a long stick of cubes and brainstormed on what to call it. Someone suggested Kevin, because Kevin Garnett wears the number 5. 10

10 means 1 Kevin and no extras

We then added some more loose (not snapped together) cubes and got

Kevin ounce 11 Kevin dice 12 Kevin trice 13 Kevin quartz 14

and then of course we had to snap them together when we got the next cube, so we had dice Kevin 20

Eventually we had 5 Kevins, and those had to be grouped together, and we got a new place value, which we voted to called Garnett 100

So we had a lot of numbers like

Garnett trice Kevin dice 132

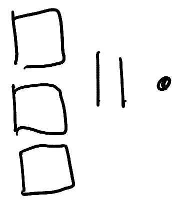
and

Dice Garnet Kevin ounce. 211

100 means 1 Garnett, no Kevins and no extras

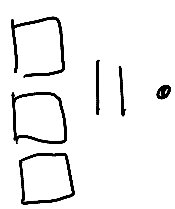
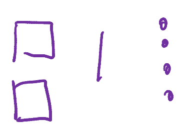
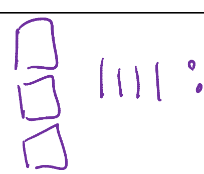
Fill in the table

This was practice on Tuesday

		
		214
Trice Garnett quartz Kevin dice		

Fill in the table

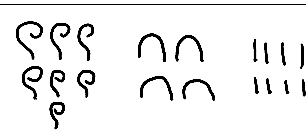
This was practice on Tuesday

Trice Garnett dice Kevin ounce		321
Dice Garnett Kevin quartz		214
Trice Garnett quartz Kevin dice		342

Things to learn:

- Learning number names and how everything fits together is hard. Kindergarteners are doing hard work!
- Patterns help out a lot. The fact that 2 Kevins is called "dice Kevin" makes your life a lot easier than if it were called "devin", and if we had a new name for the number that came 2 after Kevin and we called it "kevid" instead of "Kevin dice" it would take more time to learn. Notice that English *doesn't have* all of those nice patterns! The number that comes two after 10 isn't ten-two it's "twelve", and two tens isn't called "two tens" it's called "twenty". It takes a lot of brain space to remember all of those extra number names.
- FYI—the Chinese do this better than we do. They have words that are unique like "twelve" and "twenty" that are the traditional names, and that get used in traditional places (kind of like we use roman numerals on clocks, but not when we're adding and subtracting), but in school, learning math, everyone uses a second, simplified, set of number words, so 12 is "ten two" and 20 is "two-ten". That means kids don't have to spend as long learning number names for counting, and they get started on using numbers to solve problems sooner.

Non-positional number systems:
Used to look fancy (clocks, outlines)
(and by people who are dead—ancient Romans and Egyptians etc.)

Roman	DCCXLVIII
Egyptian Numerals	
Greek Numerals	$\psi\mu\eta$'

Our Hindu-Arabic number system (which we adopted in the 13th century from the Arabs who adapted it from the Hindu) is a positional base 10 number system. Positional means that the position of the numeral tells you its value (does it stand for a number of hundreds, tens or ones?) and that the same numeral in a different place represents a different value. Many earlier number systems were not-positional. Hundreds, tens and ones all had different symbols.

Who uses positional number systems with other bases?

241_5 241_{five}

Sometimes we use a subscript to indicate the base

Base 5:

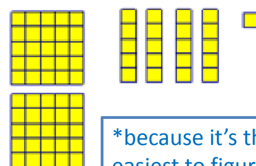
"Dice Garnetts quartz Kevins ounce"

"Two-four-one base 5"

"2 twenty-fives 4 fives and 1"

Three correct ways of reading this number out loud

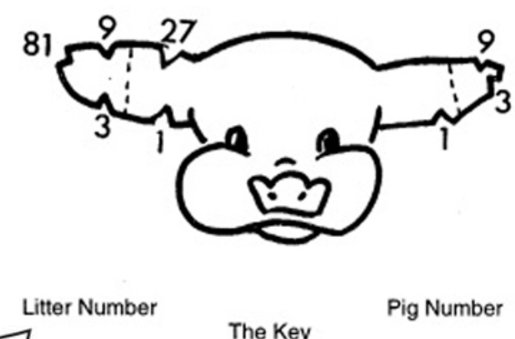
Used primarily by *math teachers**, though it's occasionally helpful when making change

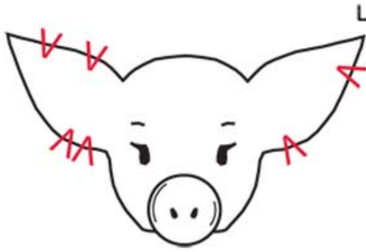


*because it's the easiest to figure out

The question I think I've answered about 6 times now:
Q: Are you suggesting I teach second graders to count and add in base 5?
A: No. I want you to experience learning how to count and add in base 5, because it will give you insight into the sorts of thinking and learning experiences second graders need to have to use base 10.

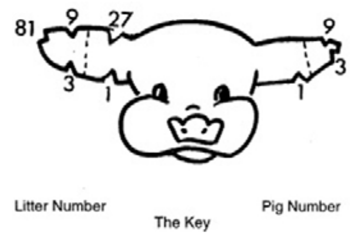
Pig farmers use base 3 (notching ears)

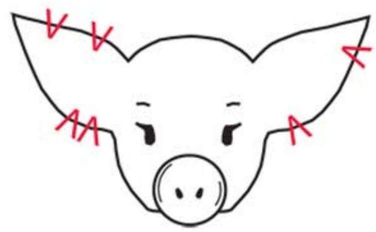




Which pig is this?

Pig farmers use base 3 (notching ears)



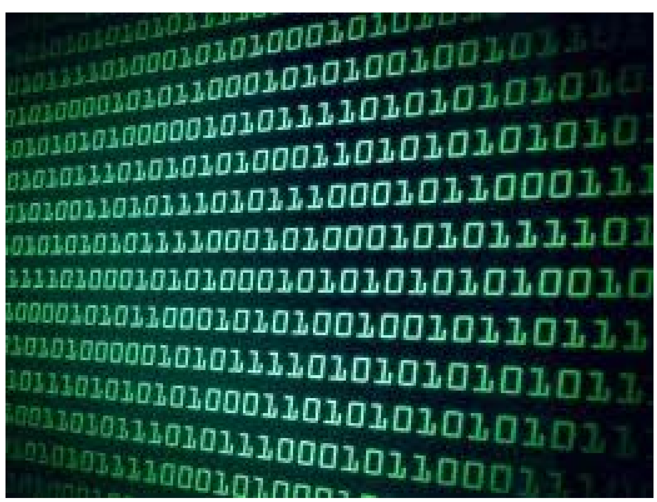


The litter number is 9+27+1+1=38

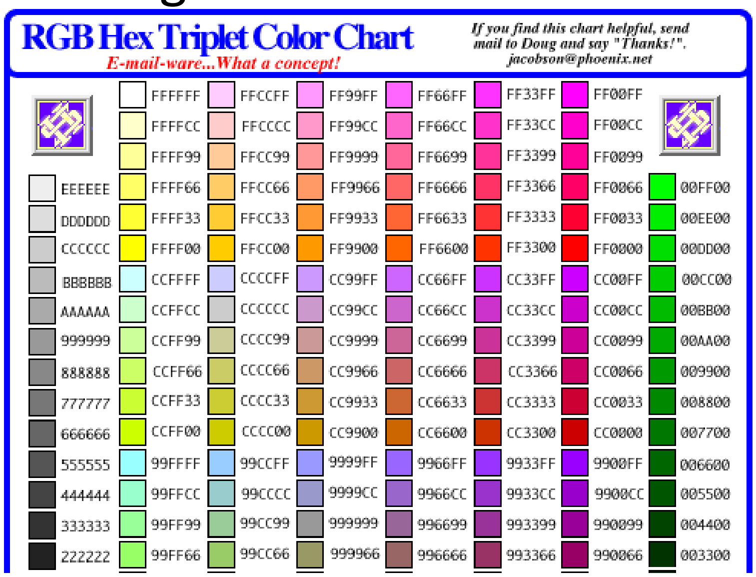
The pig number is 1+3=4

This is pig 38-4

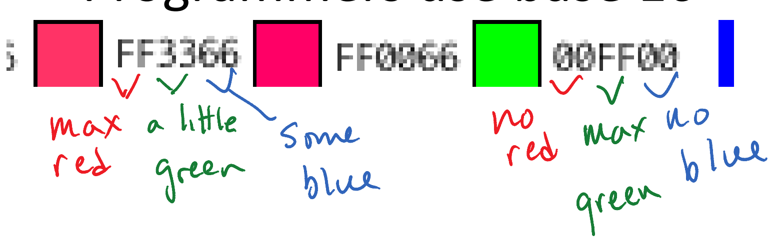
Computers use base 2:



Programmers use base 16



Programmers use base 16



Base 16 is also called hexidecimal. If you were counting in base 16 it would look like:
1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, …
Programmers really do learn and use base 16. They don't use it for everything, but there are times where it's the best choice.

Store stockers use base 12 (though they probably don't realize that they do)

Easter Punch Balls
IN-17/8
~~$6.00~~ $4.99
Per Dozen ← 12
3.5 out of 5

Colorful Bright Easter Eggs
IN-5/912
~~$10.00~~ $8.00
144 Piece(s) ← 12 twelves
4.1 out of 5

How much in base 10?

- 3 quarters, 4 nickels and 2 pennies? ← disguised base 5

  $3\times25 + 4\times5 + 2 = 75+20+2 = 97$

- 3 gross, 5 dozen and 11 ← disguised base 12

  $3\times144 + 5\times12 + 11 = 503$

- 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12 ← hexidecimal

  1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

How place values works

base 5:

125 (= 5×25)	25 (= 5×5)	5	1
2	3	1	4_5

base 10 version: $= 2\times125 + 3\times25 + 1\times5 + 4 = 334_{10}$

base 12:

144 (= 12×12)	12	1
5	3	7

base 10 version: $= 5\times144 + 3\times12 + 7 = 763$

base 16:

4096 (= 16×256)	256 (= 16×16)	16	1
2	5	A	E

base 10 version: $= 2\times4096 + 5\times256 + 16\times16 + 14 = 9646$

base 12:

144 (= 12×12)	12	1
5	3	7

base 10 version: $= 5\times144 + 3\times12 + 7 = 763$

Shopkeepers do this all the time. The make purchases in dozens and grosses, but they are not base 12 natives, and they convert amounts to base 10 right away

Programmers almost never do this. They become immersed in base 16, and it seems pointless to convert to base 10 except when talking to non-geeks. What happens in base 16 stays in base 16

base 16:

4096 (= 16×256)	256 (= 16×16)	16	1
2	5	A	E

base 10 version: $= 2\times4096 + 5\times256 + 16\times16 + 14 = 9646$

Base 5 to base 10:

Base 5 place values

125's	25's	5's	1's

341_5

$= 3 \times 25 + 4 \times 5 + 1$

$= 75 + 20 + 1 = 96$

How many gross, dozen and units?

(gross = 144, dozen = 12)

- 29? $2 \times 12 = 24$, $24 + 5 = 29$: 2 doz & 5 (25_{12})
- 156? $156 - 144 = 12$ — 1 gross & 1 dozen (110_{12})
- 1037?

$144 \times 5 = 720$; $720 + 144 = 864$

$864 + 144 = 1008$ — 7 gross

$1037 - 1008 = 29$ = 2 doz + 5

7 gross, 2 doz & 5

How many in base 5?

125	25	5	1

- 16? $3 \times 5 = 15$ $15 + 1 = 16$ … 31_5
- 27? $27 = 25 + 2$ ∴ 102_5
- 63? $2 \times 25 = 50$; $63 - 50 = 13$; $13 = 2 \times 5 + 3$ 223_5
- 179? $179 - 125 = 54$
  $54 = 2 \times 25 + 4$ 1204_5
- 286?

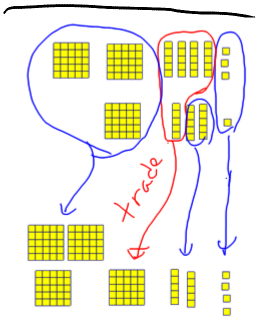
$2 \times 125 = 250$

$286 = 250 + 36 = 2 \times 125 + 1 \times 25 + 2 \times 5 + 1$

$= 2121_5$

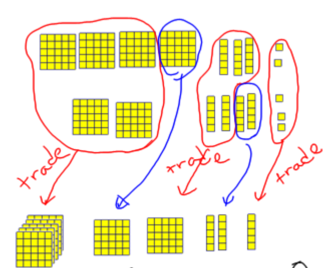
Add in base 5 without converting to base 10!

$$\begin{array}{r} 243 \\ +\ 131 \\ \hline \end{array}$$



4 2 4

$$\begin{array}{r} 432 \\ +\ 243 \\ \hline \end{array}$$



1 2 3 0

Subtract in base 5 without converting to base 10

```
  4 1 3        4 2 1
- 2 3 1      - 1 3 4
-------      -------
```

Homework (due Friday)

Convert from base 5 to base 10:

1. 423   2. 1324   3. 1041

Convert from base 10 to base 5:

4. 28   5. 82   6. 341

Add in base 5 without converting to base 10

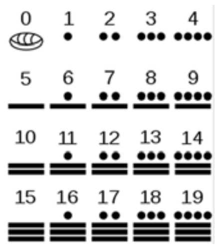
7. 243+142

Subtract in base 5 without converting to base 10

8. ~~421 - 143~~ not due Friday

People who are dead who used a positional number system that was not base 10: Base 20

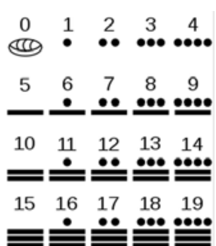
K'inich Yax K'uk' Mo'
(and lots of other ancient Mayans)



This image is from the Dresden Codex →
http://bibliodyssey.blogspot.com/2010/02/oldest-book-from-americas.html

this page is about eclipses

People who are dead who used a positional number system that was not base 10: Base 20



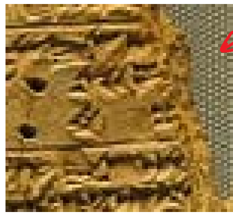
16000's
400's
20's
1's place

1 x 16000
+ 9 x 400
+ 15 x 20
+ 8 x 1

19908

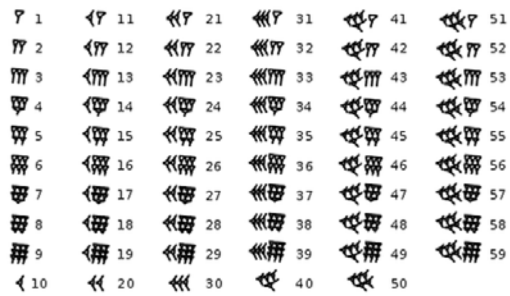
↑ base 10 version of this number

Ancient Babylonians and all astronomers up until the end of the Roman Empire (maybe later) used base 60:



clay tablets

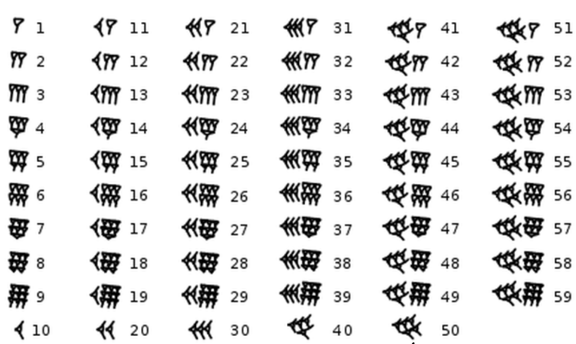
The Venus Tablet (17th century BC) – When planet Venus rises and sets



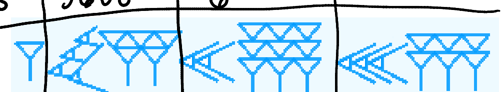
Hammurabi (Babylonian ruler)

Hipparchus (A Greek astronomer 190-120 BC)

The Babylonian number system was used by astronomers everywhere, because it was the only positional number system around, and it was good for the large numbers astronomers needed.

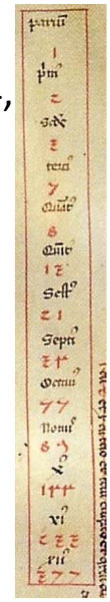


60x3600 = 216000	60x60 = 3600	60	1



base 10: 216000 + 45x3600 + 29x60 + 36 = 379,776

We have Leonardo Pisano Bonacci (Fibonacci) to thank for introducing Hindu-Arabic numbers to Europe (1170-1250) in his book Liber Abaci (1202)



Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34,

You've probably heard of Fibonacci numbers (aka the Fibonacci sequence), but that bit of trivia was just an appendix in his book **Liber Abaci** (book of numbers or book of computation). Most of the book was showing people the Arabic numbers he had learned from his Arab-empire tutor while his father was working in northern Africa. Look! (it said) these numbers are really useful! They're so much more efficient than Roman numbers. Here's how you add and subtract and multiply and solve problems using these numbers. Try it! It's great! So every time you write 24 instead of XXIV, you should be thanking Fibonacci (and, of course, rather a lot of Arab and Hindu mathematicians—but they didn't write an immensely popular book, so we don't know exactly who to give credit to).