Math 246 review for test 2: Number and operations

Addition and subtraction invented algorithms:

- know how to add on an open number line
- know how to subtract by adding up on an open number line (using numbers that end in 0)
- know how to by adding in place values and combining
- know how to subtract using the negative numbers algorithm
- be able to explain the steps in a student algorithm (see homework sheet last problem)

1. Show **two ways** of doing each calculation that are **different from the standard algorithm.** Know how to show at least one strategy for each on an open number line.

a.36 + 29

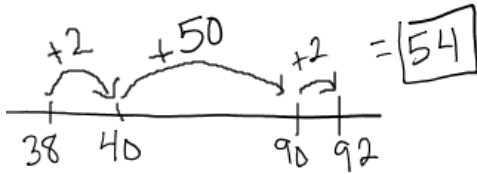a. open number line solutions:
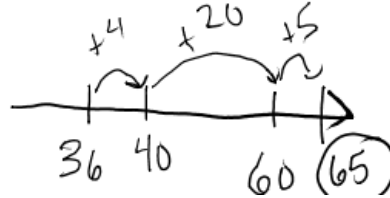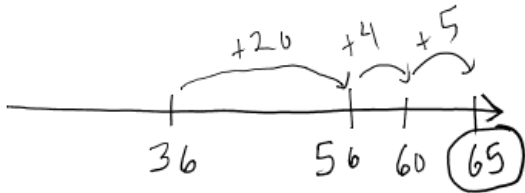
add in place values and combine:

30+20=50
6+9=15
50+15=65

b. 92 – 38

Adding up on an open number line (using numbers that end in 0)	negative numbers algorithm 90-30=60 2-8 = -6 60-6=54

Addition and subtraction standard algorithm

- Explain in words, without using "carry" or "borrow" particular steps in an addition or subtraction problem
- Show with manipulative pictures and numbers how to do part of an addition or subtraction problem.
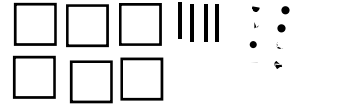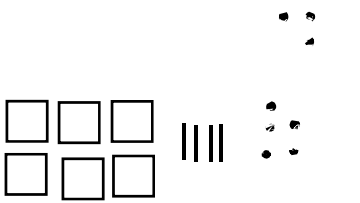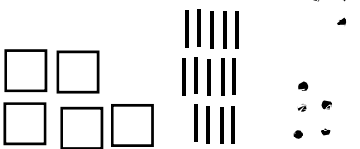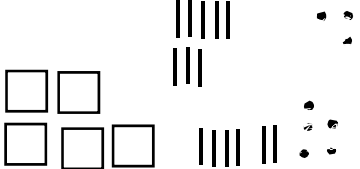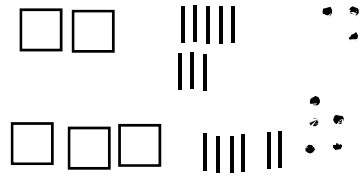
2. Fill in the missing step, and explain both steps:

Trade 1 hundred for 10 tens. Write above the 100's place to show that there are now only 3 hundreds, and record 10 above the tens place that there are now 10 tens.

Then, trade 1 ten for 10 ones. Write above the tens place that there are now only 9 tens, and write above the ones place that there are now ten more ones, for a total of 13 ones.

3. For each step, fill in the missing manipulative picture, number work or explanatory sentence:

(blocks)	number work	explanation
	6 4 8 − 2 8 3	Set out blocks to show 6 hundreds, 4 tens and 8 ones.
	6 4 8 − 2 8 3 5	I can separate out 3 ones from the 8 ones. There are 5 ones left when I am done, so I write 5 in the ones place of the answer
	$\overset{5}{\cancel{6}}$ 14 8 − 2 8 3 5	I need more tens, so I trade 1 hundred for 10 tens. I change 6 hundred to 5 hundred in my number work, and I change 4 tens to 14 tens.
	$\overset{5}{\cancel{6}}$ 14 8 − 2 8 3 6 5	I take away 8 tens from 14 tens, and I have 6 tens left. I write 6 in the tens place of the answer.
	$\overset{5}{\cancel{6}}$ 14 8 − 2 8 3 3 6 5	Take away 2 hundreds from 5 hundreds. There are 3 hundreds left, so write 3 in the hundreds place.

Solve and explain solutions to division problems using manipulatives, long division and scaffolding division.

6. Solve by scaffolding, drawing out the groups: $6\overline{)2780}$

| 7. Draw what the manipulatives would look at this point in the long division algorithm.

 | $\begin{array}{r} 2\ 4 \\ 6\overline{)\ 1489} \\ -12 \\ \hline 28 \\ -24 \\ \hline 49 \end{array}$ | Explain what each of the numbers represents in the manipulatives and the problem:

a. What is 6?
6 groups

b. What is 24?
240 in each group

c. What is 49?
49 ones waiting to be shared. |

Multiplication
- draw a diagram and write an explanation for the commutative law
- draw a diagram and write an explanation for the distributive law
- draw a base 10 blocks diagram for a 2 digit by 2 digit multiplication problem, compute the product using both the expanded and the standard algorithm, and label or color code it to show how the algorithms correspond.
- multiply using the standard algorithm and the expanded algorithm

8. Explain with words and a diagram why it works and makes sense that 4 × 6 = 6 × 4.

← 4 groups
6 in each group
4 × 6

6 groups
4 in each group · 6×4

The same array can be grouped to show 4x6 and grouped another way to show 6x4. That means 4 × 6 and 6 × 4 must be the same amount.

What is the name of this property? Commutative

9. Explain with words and a diagram why it works and makes sense that 6 × (5+2) = (6 × 5) + (6 × 2).
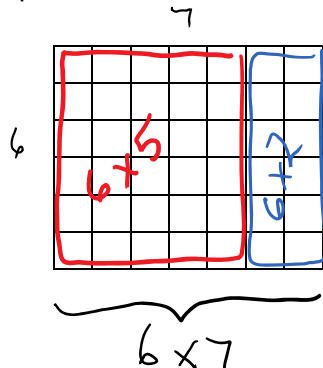
The 6 × 7 array can be split into a 6 × 5 and a 6 × 2 part, so

6 × 7 = 6 × (5+2) = (6 × 5) + (6 × 2)

What is the name of this property? Distributive

10. a. Show how to compute
$$\begin{array}{r} 5\ 4\ 8 \\ \times\ \ 3\ 7 \end{array}$$
using the standard algorithm.

b. Before computing 3 × 8 in the standard algorithm we write a 0 in the partial product. Explain why we write a 0 there.

3 × 8 is really 30 × 8 = 240, so there are 0 ones. Put 0 in the ones place because there are 0 ones.

OR

We're multiplying by a number in the tens place, so our answer is going to be numbers in the tens place or higher: that means we'll have 0 ones, so put 0 in the ones place.

c. When we computer 3 × 8 =24 on the standard algorithm, we write 4 in the tens place, and we write 2 above the tens place. Why does 4 go in the tens place, and why does 2 go above the tens place?

30 × 8 = 240. The 4 is 4 tens, so put it in the tens place.

The 2 is 2 hundreds, but we need to look ahead to the next calculation.

The next thing we multiply will be 30 × 40 which will be a number of hundreds. If we put 2 (hundred) above the 4 (tens) it will get added to the hundreds we will get when we do 30 × 40 (so it will end up in the hundreds place of the answer).

Solution to #11 on the next page

12. Write a word problem for 32 × 14

A box of crayons has 32 crayons in it. How many crayons are in 14 boxes?

Use equals signs correctly:

13. Fix the equals signs while keeping the thinking the same

| a. $86-2=84\div4=21$ rewrite this one with shorter 1-step equations

$86-2=84$

$84\div4=21$ | b. $\dfrac{1}{2}\times3\times4=\dfrac{1}{2}\times12=6\times4=24+16=40$ rewrite this one into a single complex equation. Show the steps in calculating the answer from the complex equation.

$\left(\dfrac{1}{2}\times(3\times4)\right)\times4+16=$

$\left(\dfrac{1}{2}\times12\right)\times4+16=$

$6\times4+16=$

$24+16=40$ |

14. Write down this numerical calculation (for 4x7) using correct equations:

Two 7's are 14, and another 7 is 21 and another 7 makes 28.

$2 \times 7 = 14$

$14 + 7 = 21$
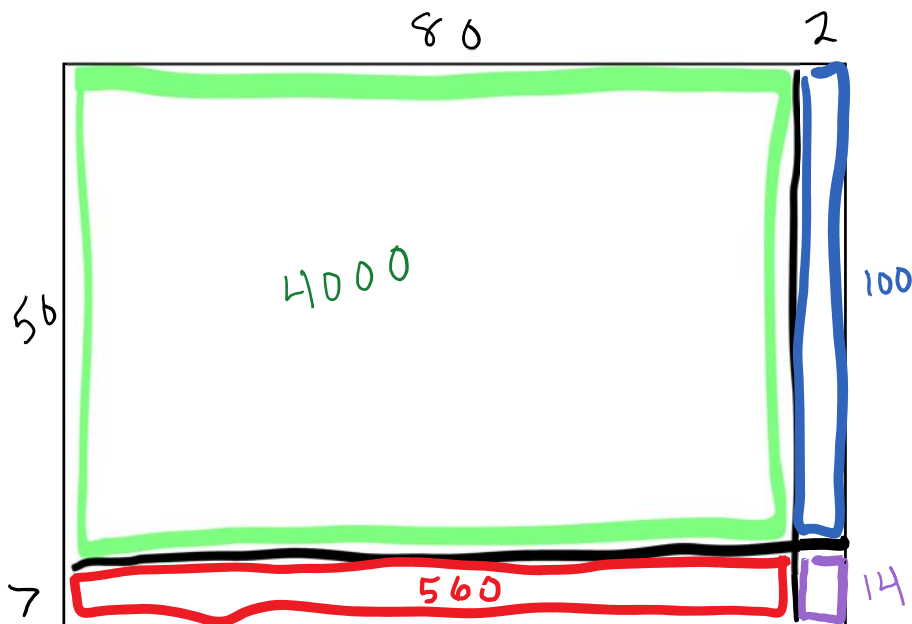
$21 + 7 = 28$

so $4 \times 7 = 28$

11. a. Sketch an array diagram for: 57 × 82
Note: I am planning to give you a grid, and to give you numbers that will fit onto the grid that I give you. With numbers this large, one would need to make a non-proportional array like this one:

* Note that a non-proportional array is just like the arrays we have been making on graph paper, except that we aren't counting and measuring to make sure all of our lengths are to scale: I drew 50 to look a little shorter than 80, and 7 to look a lot shorter than 50, but they aren't exactly proportional, and a non-proportional array is allowed to be even less to scale.

An even less proportional version of the array:

b.
```
    8 2
  ×  5 7
      1 4
    5 6 0
    1 0 0
  4 0 0 0
─────────
  4 6 7 4
```

c.
```
    8 2
  ×  5 7
    5 7 4
  4 1 0 0
─────────
  4 6 7 4
```