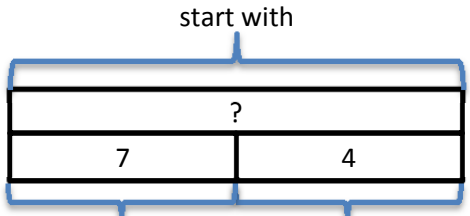
Math 246 Review 2:

1. Draw a bar diagram to show how to solve each of the following word problems, and write the associated equations:

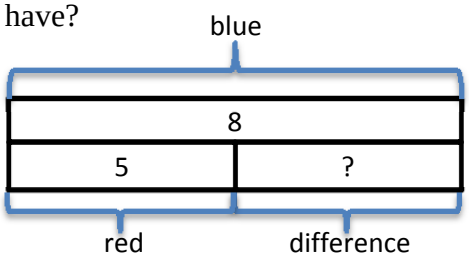
- Addition and subtraction bar diagrams need labels
- Multiplicative comparison bar diagrams need labels
- Other multiplication and division bar diagrams do not need labels
- Each bar diagram needs an addition or multiplication equation (which may be a missing part equation)
- Some bar diagrams should also have a subtraction or division equation.

a. Sandra had 4 erasers left after she gave 7 erasers to her friends. How many erasers did she start with?



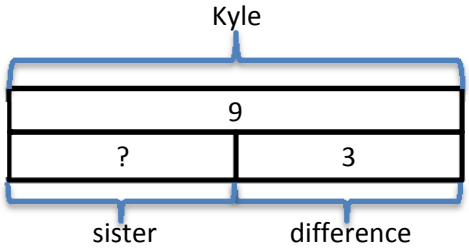
7+4=?
optional: ?-7=4

b. Marie has 8 blue balloons and 5 red balloons. How many more red balloons than blue balloons does she have?



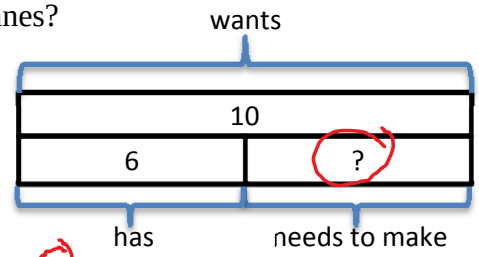
8-5=?
5+?=8

c. Kyle has 9 transformers. He has 3 more than his sister. How many does his sister have?



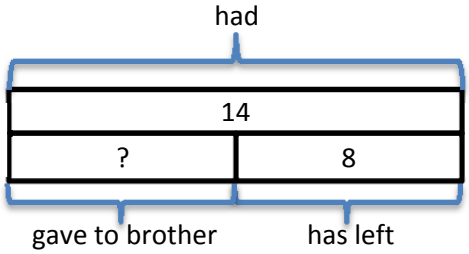
9-3=?
3+?=9

d. Amanda has 6 origami cranes. How many more does she need to make to have 10 origami cranes?



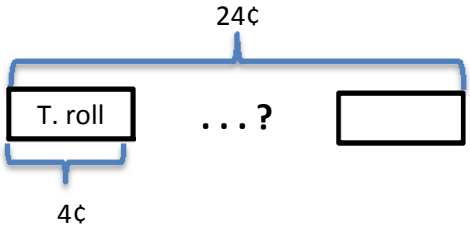
10-6=?
6+?=10

e. Paul had 14 cookies. He gave some to his brother, and now he has 8. How many cookies did he give to his brother?



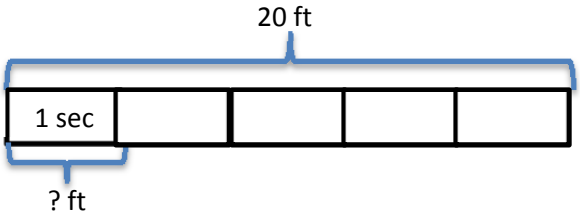
14-8=?
8+?=14

f. A tootsie roll costs 4¢. Ross has 24¢. How many tootsie rolls can he buy?



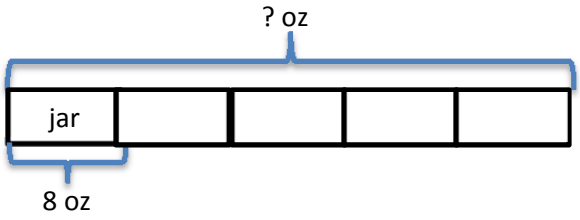
24 ÷ 4 = ?
4 × ? = 24

g. A toy train can go 20 feet in 5 seconds. How many feet can it go in one second?



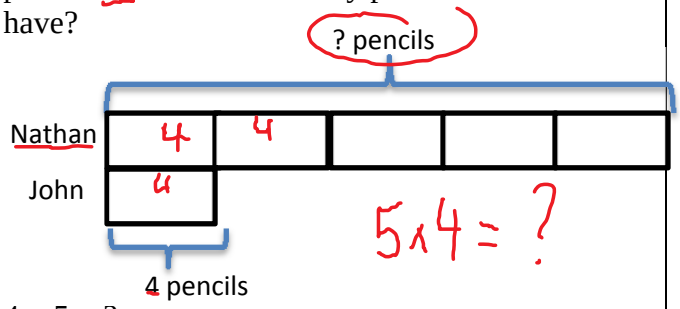
20 ÷ 5 =?
5 × ? = 20

h. A Jar of jam has 8 ounces of jam in it. How many ounces of jam are in 5 jars?



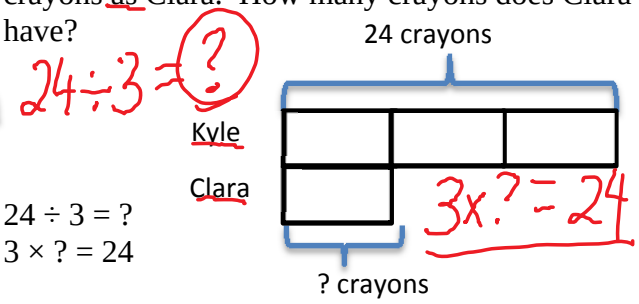
8 × 5 = ?

i. John has 4 pencils. Nathan has 5 times as many pencils as John. How many pencils does Nathan have?



4 × 5 = ?

j. Kyle has 24 crayons. He has 3 times as many crayons as Clara. How many crayons does Clara have?



24 ÷ 3 = ?
3 × ? = 24

2. Show **two ways** of doing each calculation that are **different from the standard algorithm**
a.36 + 29 c. 92 - 38
(many correct solutions, including open number line, expanded, rounding and negative number solutions)

3. Explain (using appropriate base 10 language) the following two steps in the standard subtraction algorithm:

$$\begin{array}{r} 6\ 2\ 8 \\ -\ 2\ 9\ 3 \\ \hline 5 \end{array} \Rightarrow \begin{array}{r} 5\ 12 \\ \cancel{6}\ \cancel{2}\ 8 \\ -\ 2\ 9\ 3 \\ \hline 5 \end{array} \Rightarrow \begin{array}{r} 5\ 12 \\ \cancel{6}\ \cancel{2}\ 8 \\ -\ 2\ 9\ 3 \\ \hline 3\ 5 \end{array}$$

In the first step, I trade a hundred for 10 tens. Write down that there are now 5 hundreds (cross out 6), and there are now 12 tens (instead of 2).
In the second step, I take away 9 tens from 12 tens, which leaves 3 tens. Write 3 in the tens place of the answer.

4. Show how to solve each of these using the appropriate expanded algorithm:
a. 478 + 394 b. 723 – 186 c. 246 × 87

$$\begin{array}{r} 478 \\ +\ 394 \\ \hline 12 \\ 160 \\ 1200 \\ \hline 1372 \end{array}$$

$$\begin{array}{r} 600 \\ -\ 100 \\ \hline 500 \end{array} + \begin{array}{r} 110 \\ -\ 80 \\ \hline 30 \end{array} + \begin{array}{r} 13 \\ -\ 6 \\ \hline 7 \end{array} = 537$$

$$\begin{array}{r} 246 \\ \times\ 87 \\ \hline {\scriptstyle 1}42 \\ {\scriptstyle 1}280 \\ 1400 \\ 480 \\ {\scriptstyle 1}3200 \\ 16000 \\ \hline 21322 \end{array}$$

5. Show how to solve the following problem using scaffolding division in a way that uses easier multiplication facts than the most efficient solution:
8081 ÷ 12

$$\begin{array}{r} 673R5 \\ 12\overline{)8081} \end{array}$$

$$\begin{array}{r|l} 4800 & 400 \\ \hline 3281 & \\ 2400 & 200 \\ \hline 881 & \\ 480 & 40 \\ \hline 401 & \\ 360 & 30 \\ \hline 41 & \\ 36 & 3 \\ \hline 5 & 673 \end{array}$$

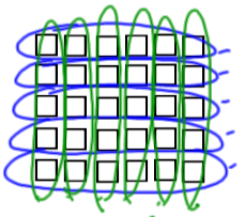
6. a. Explain how knowing the commutative law of multiplication helps children learn the multiplication facts

It means that if a student knows one multiplication fact already (6x5), then they don't have to learn the turn-around fact (5x6) if they know the commutative law. It also lets them choose between two strategies/interpretations (5 groups of 6 or 6 groups of 5) to choose the one that they know best how to find the product.

b. Draw a diagram and write an (in words) explanation that shows why the commutative law of multiplication makes sense (you may show it for a specific example)

version 1:

Commutative law:
axb=bxa
6x5=5x6



5x6 =
5 groups of 6

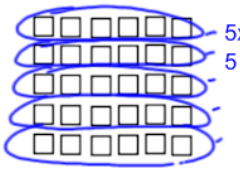
6x5=6 groups of 5

6 groups of 5 and 5 groups of 6 are two ways of counting the same squares, so they are equal.

version 2:

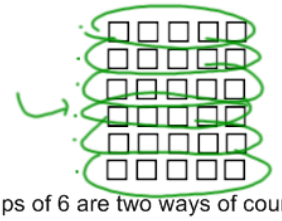
Commutative law:
axb=bxa
6x5=5x6

These two arrays have the same number of squares in them because one is just turned on its side to make the other.



5x6 =
5 groups of 6

6x5=6 groups of 5



6 groups of 5 and 5 groups of 6 are two ways of counting the same amount of squares, so they are equal.

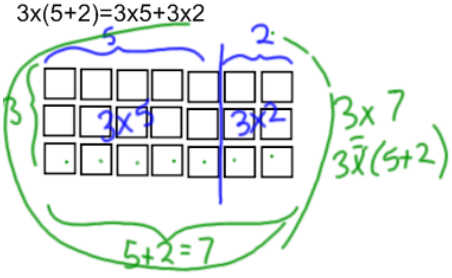
7. a. Explain how knowing the distributive law of multiplication over addition helps children learn the multiplication facts

Children can break an unknown multiplication fact down into two known multiplication facts and add them together to find the total product

b. Draw a diagram and write an (in words) explanation that shows why the commutative law of multiplication makes sense (you may show it for a specific example)

version 1:

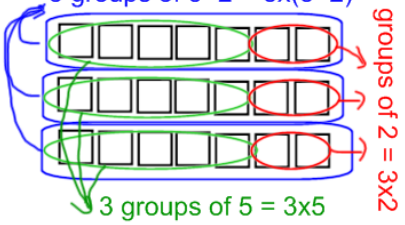
Distributive Law
a(b+c)=ab+ac
3x(5+2)=3x5+3x2



The array for 3x(5+2) can be split into two smaller arrays: 3x5 and 3x2, so the number of squares in 3x(5+2) is the same as the squares in 3x5+ the squares in 3x2.

version 2:

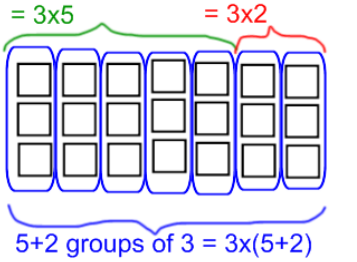
Distributive Law
a(b+c)=ab+ac
3x(5+2)=3x5+3x2



3 groups of 7 is the same amount as 3 groups of 5 and 3 groups of 2 because each 7 can be split into 5 and 2

version 3:

Distributive Law
a(b+c)=ab+ac
3x(5+2)=3x5+3x2



7 groups of 3 is the same as 5 groups of 3 and 2 more groups of 3

8. Write **a.** a partition and **b.** a measurement division word problem for 36 ÷ 4.

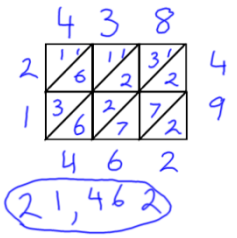
a. A partition division problem tells you the number of sets. For example "I have 36 cookies that I want to put equally into 4 bags. How many cookies should go in each bag?"

b. a measurement division problem tells you how many is in each set. For example: "I have 36 cookies. I want to put 4 cookies into each bag. How many bags can I fill?"

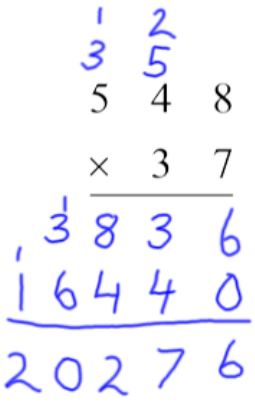
9. Write a word problem for 32 × 14

Example: A shirt costs $32. How much do 14 shirts cost?

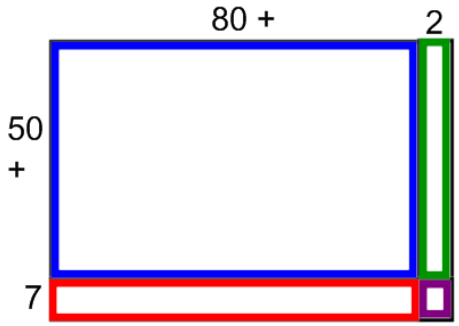
10. Show how to compute 438 × 49 using the lattice algorithm



11. a. Show how to compute $\begin{array}{r} 5\ 4\ 8 \\ \times\ 3\ 7 \\ \hline \end{array}$ using the standard algorithm.

a. Compute with the standard algorithm	b. Before computing 3 × 8 in the standard algorithm we write a 0 in the partial product. Explain why we write a 0 there.	c. When we computer 3 × 8 =24 on the standard algorithm, we write 4 in the tens place, and we write 2 above the tens place. Why does 4 go in the tens place, and why does 2 go above the tens place?
	*3 is 3 tens, so the product should go in the tens place (24 tens), so we put a 0 in the ones place.*	*The 3 is 3 tens, so 24 is 24 tens, so 4 should go in the tens place (because it's 4 tens). The 2 is 20 tens = 200, so it needs to get added in the next place value. I'll get the next place value (hundreds) when I multiply 30×40, so I should put 2 above the 4 to make it easy to add on.*

12. a. Sketch an array diagram for: 57 × 82



Notes about the array:
One side must have length 80+2
The other side must have length 50+7
The array must be subdivided into 4 parts where the numbers split between tens and ones.

The lengths do not have to be to scale or even approximately to scale unless I provide a grid.

expanded:

standard:

note that you could optionally choose to double color the 7 in 574 as red and purple, and the 1 in 4100 as blue and green.

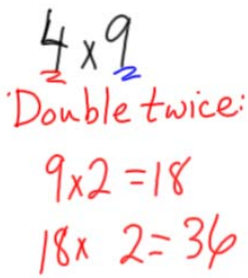
13. a. Write a division problem with a remainder where the answer that makes sense is the quotient
This needs to be a problem where it would make sense to discard the remainder: "I have 25 cookies. 4 cookies can fit in a bag. **How many bags can I fill?**"

b. Write a division problem with a remainder where the answer that makes sense is the quotient+1
This needs to be a problem where you have to keep and make a group for the remainder: "There are 25 children. 4 children can ride in each car. How many cars do we need **to take all of the children** to the zoo?"

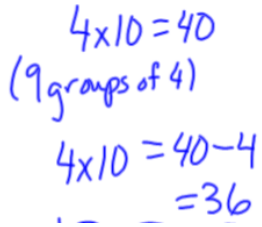
14. Analyze and explain an error pattern or an alternate algorithm for addition, subtraction, multiplication or division.

15. Show two ways of figuring out 4 × 9 using efficient strategies.
An efficient strategy is faster than skip counting (4+4+….+4), but is not just a memorized answer.

Focusing on the 4, and thinking of the problem as 4 groups of 9:



Focusing on the 9, and thinking of the problem as 9 groups of 4:

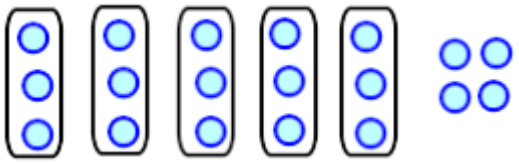


(10 groups of 4 – 1 group of 4)
or use the 9's pattern: 4x9 will have 3 as the tens digit (one less than 4). The tens and ones digits will add to 9: 3+ 6 =9 so 36.

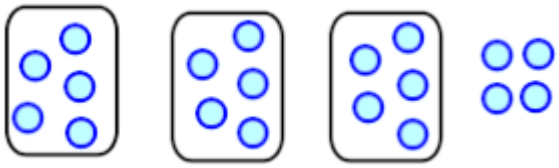
16. Show two ways of figuring out 6 × 7 using efficient strategies.

Thinking 6 groups of 7, and breaking 6 into 5 and 1:	6 groups of 7, and 6=2+2+2	6 groups of 7 and 6=3+3	7 groups of 6 and 7=5+2
6x7 = Six sevens 5x7=35 five sevens 35+7=42 5+1 sevens	2 sevens 3 times 14 14 14 28 42	3x7 + 3x7 21 + 21 42	6x7 Seven sixes five sixes=5x6=30 +2 sixes = 12 42

17. a. Show what a direct modeling type picture (so you could count each object to find the answer) of a partitive division solution for 19 ÷ 5 would look like.
partitive means 5 is the number of sets



b. Show what a direct modeling type picture (so you could count each object to find the answer) of a ~~partitive~~ measurement division solution for 19 ÷ 5 would look like. (one of these should have been measurement).
measurement means the 5 is the size of each set



18. In the standard division algorithm as done with base 10 blocks:

```
   458
4)1834
  16
  ---
   23
   20
  ---
    34
    32
  ---
     2
```

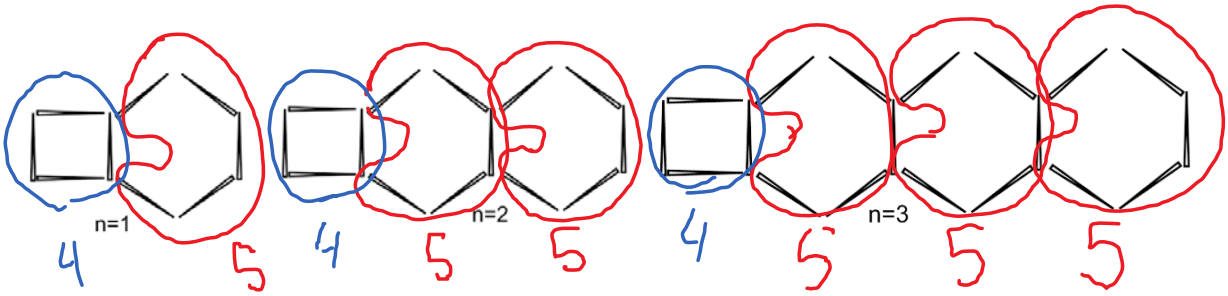
a. What does 5 represent?
5 is the number of tens in each group

b. What does 23 represent?
23 is the number of tens left at this point in the process (waiting to be shared into the groups)

c. What does 20 represent?
20 is the total number of tens put into groups

d. What does 34 represent?
34 is the number of ones left (waiting to be put into groups)

19. In this pattern, we are looking for the number of toothpicks it takes to build a stage *n* design. Find and carefully explain an equation for the pattern:



Things to show: how to group the objects in similar ways in each pattern.(Label how many are in the groups)
At each step (n) there are n sets of 5 toothpicks that make most of a hexagon, so there are 5n toothpicks in the red circled part.
Explain in a sentence the repeated sets (__ sets of __)
At each step there are 4 extra toothpicks that make a square (the blue circled part).
Explain in a sentence the extras that are the same at each step.
In all there are 5n+4 toothpicks at step n
Write the whole equation down. (If you color code things you are more likely to get full credit even if some detail is missing from your sentences.)