**Teachers know the subjects they are teaching.**   
The teacher understands the central concepts, tools of inquiry, and structures of the disciplines she or he teaches and can create learning experiences that make these aspects of subject matter meaningful for pupils.

*Identify the conceptual models relevant for the unit. Describe how what children do with these models fits into the structure of what children learn in math across grade levels.*

The central conceptual models for multiplication are repeated addition and arrays. Repeated addition is the first structure we use to understand what multiplication is and why it works. Although in later grades, multiplication becomes more connected with areas, even fraction multiplication has to be consistent with, and include the repeated addition way of thinking about multiplication. Arrays (or rectangles) can be seen at this level as a way of organizing repeated addition to see patterns (like the commutative law). Later, arrays become one of the organizing themes that defines multiplication for fractions and decimals.

The central conceptual models for division are fair shares (dividing into a known number of equal sets), repeated subtraction (dividing into equal sets of a given size), and division as the inverse of multiplication. At this level, we think of division in all three of these ways, but doing division mostly with counting numbers less than 100. Experience with solving division of both fair shares and repeated subtraction types helps children understand when division is useful for solving problems. Experience with thinking of division as the inverse of multiplication helps children learn efficient ways to find answers to division problems. A deep understanding of these ways of thinking of division can be helpful to children in later grades in both problem solving situations, and in understanding division of fraction and decimals, and in solving algebra problems.

*Identify where in the unit children work with these central concepts, and discuss how the lessons help children make sense of these concepts and understand them more clearly. Identify some*

*places where children build new knowledge by problem solving and reasoning using central conceptual models and tools.*

In the first lesson, children are representing multiplication problems with repeated addition direct modeling diagrams, with repeated addition in numbers and bar diagrams, and with multiplication notation and rectangle diagrams. The goal of this lesson is for children to be able to go back and forth between different representations of multiplication, and realize that they can solve a multiplication problem in any of several different ways. If children can see how the different ways of representing multiplication all mean the same thing, then they will have a better understanding of what multiplication is and how multiplication problems can be solved.

In the second lesson children are investigating patterns with rectangle (array) diagrams. The goal of this lesson is for children to discover and articulate the commutative law of multiplication. Rectangle diagrams were chosen as the investigation tool for this lesson because it’s easy to see how the lengths of the sides of the rectangles correspond to the multiplier and the multiplicand in a multiplication problem. The commutative law is both an important key idea in understanding multiplication and a key tool used in computing multiplication. In this lesson children both conceptually represent and discover the commutative law, and they practice using it to help them compute products.

Lessons 3 and 4 review ideas of division and connect division solutions by direct modeling and repeated subtraction with missing number multiplication problems. One goal of this lesson is to introduce using division as the inverse of multiplication to solve division problems.

In lessons 5-7, children draw bar diagrams for multiplication and partition division (division into equal parts where the number of parts is known) problems. The emphasis is on recognizing multiplication and division in context, with continued discussion of different ways to numerically solve problems using the relationships between different models of multiplication and division.

In lessons 8-12, children first learn patterns for multiplying by multiples of 10, 100 and 1000 by using repeated addition with base 10 materials. Then children learn to break problems where 2 and 3 digit numbers are multiplied by a 1 digit number into place value problems, and combine them to find the total product.

Lessons 13-15 introduce division with remainders by first solving division problems (partition division—breaking into a known number of groups) by direct modeling with base 10 materials, and then with the numerical algorithm, by making the connection to division as the inverse of multiplication.