f: Vector space $\to$ Vector space

$f: \mathbb{R}^2 \to \mathbb{R}^2$

Basis $\{(1,0), (0,1)\}$

$$\underbrace{(x,y) = x(1,0) + y(0,1)}_{\to \text{ Linear combinations}}$$

Linear Transformation

$(1,0) \to \vec{u}$

$(0,1) \to \vec{v}$

$$\underbrace{(x,y) \to x\vec{u} + y\vec{v}}_{\text{Linear Transformation}}$$

$f(1,0) = (2,3)$

$f(0,1) = (7,12)$

$f(x,y) = x(2,3) + y(7,12)$

$= (2x+7y, 3x+12y)$

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+7y \\ 3x+12y \end{pmatrix}$$

$$\begin{pmatrix} 2 & 7 \\ 3 & 12 \end{pmatrix}$$

$g(1,0) = (-4,5)$

$g(0,1) = (10,9)$

$g(x,y) = x(-4,5) + y(10,9)$

$= (-4x+10y, 5x+9y)$

$$g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4x+10y \\ 5x+9y \end{pmatrix}$$

$$\begin{pmatrix} -4 & 10 \\ 5 & 9 \end{pmatrix}$$

$$g \circ f(x,y) = g(2x+7y,\ 3x+12y)$$

$$= (-4(2x+7y)+10(3x+12y),\ 5(2x+7y)+9(3x+12y))$$

$$= ((-4\cdot 2+10\cdot 3)x + (-4\cdot 7+10\cdot 12)y,\ (5\cdot 2+9\cdot 3)x + (5\cdot 7+9\cdot 12)y)$$

$$g \circ f\begin{pmatrix} x \\ y \end{pmatrix} = g\begin{pmatrix} 2x+7y \\ 3x+12y \end{pmatrix}$$

$$= \begin{pmatrix} (-4\cdot 2+10\cdot 3)x + (-4\cdot 7+10\cdot 12)y \\ (5\cdot 2+9\cdot 3)x + (5\cdot 7+9\cdot 12)y \end{pmatrix}$$

$$\begin{pmatrix} -4 & 10 \\ 5 & 9 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ 3 & 12 \end{pmatrix} \rightarrow \begin{pmatrix} (-4\cdot 2+10\cdot 3) & (-4\cdot 7+10\cdot 12) \\ (5\cdot 2+9\cdot 3) & (5\cdot 7+9\cdot 12) \end{pmatrix}$$

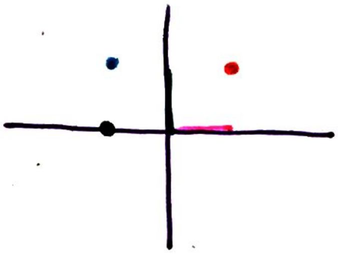
matrix multiplication composes Linear transformation

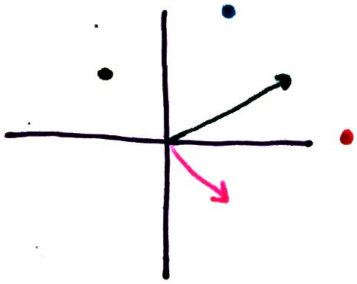
Linear transformation

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

point $(-1,0)$

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$





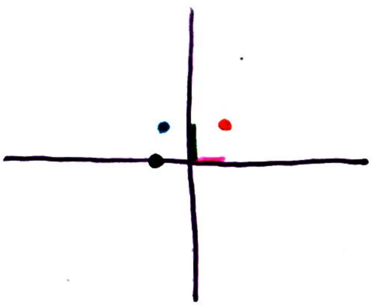
which vector stretches the most?

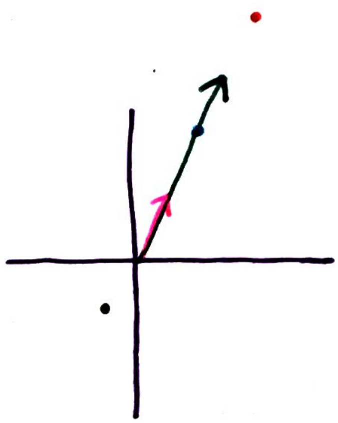
- sometimes linear transformations collapse

$$\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

- does not span

det = 0





$$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$