Some useful definitions from a Heath Algebra 2 book

- A *rational number* is a number that can be expressed as the quotient of two integers. Rational numbers can be expressed as repeating decimals or terminating decimals.
- Real numbers that are not rational numbers are called *irrational numbers*. Irrational numbers can be expressed as nonterminating, nonrepeating decimals.
- Order of operations
  1. Evaluate inside grouping symbols first.
  2. Evaluate powers and roots next.
  3. Do multiplications and divisions in order from left to right.
  4. Do additions and subtractions in order from left to right.
- Trichotomy property

  For all real numbers a and b, one and only one of the following statements is true:

  $$a < b, \quad a = b, \quad a > b.$$
- Transitive property of order

  For all real numbers a, b, and c,

  $$\text{if } a < b \text{ and } b < c, \text{ then } a < c.$$

Field Postulates of the Real-Number System

	Addition	*Multiplication*
Closure Property	For all real numbers a and b, $a + b$ is a real number.	For all real numbers a and b, ab is a real number.
Commutative Property	For all real numbers a and b, $a + b = b + a$.	For all real numbers a and b, $ab = ba$.
Associative Property	For all real numbers a, b, and c, $(a + b) + c = a + (b + c)$.	For all real numbers a, b, and c, $(ab)c = a(bc)$.
Identity Property	There is a real number 0, such that for each real number a, $a + 0 = 0 + a = a$.	There is a real number 1, such that for each real number a, $1 \cdot a = a \cdot 1 = a$.
Inverse Property	For each real number a, there is a real number $(-a)$ such that $a + (-a) = -a + a = 0$.	For each real number a, $a \neq 0$, there is a real number $\left(\frac{1}{a}\right)$ such that $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$.
Distributive Property	For all real numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$.	

- Definition of subtraction [1–5]

  For all real numbers a and b, $a - b = a + (-b)$.

- Definition of division [1–5]

  For all real numbers a and b, $b \neq 0$, $\frac{a}{b} = a\left(\frac{1}{b}\right)$.

- Properties of Equality [1–7]

Properties of Equality

Reflexive Property

For each real number a, $a = a$.

Symmetric Property

For all real numbers a and b, if $a = b$ then $b = a$.

Transitive Property

For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Substitution Property

For all real numbers a and b, if $a = b$, then a may be substituted for b in any sentence in which b occurs (or b for a in any sentence in which a occurs) without changing the truth or falseness of the sentence.

Addition Property of Equality

For all real numbers a, b, and c,

if $a = b$, then $a + c = b + c$.

Multiplication Property of Equality

For all real numbers a, b, and c,

if $a = b$, then $ac = bc$.

Example 1 Solve. $3(2x - 5) = 12 + 4x$

Solution

	$3(2x - 5) = 12 + 4x$
Use the distributive property.	$6x - 15 = 12 + 4x$
Add $-4x$ to both sides.	$2x - 15 = 12$
Add 15 to both sides.	$2x = 27$
Multiply both sides by $\frac{1}{2}$.	$x = \frac{27}{2}$

Answer The solution set of the given equation is $\left\{\frac{27}{2}\right\}$.