Abstract Algebra final practice assignment:

1. (Adapted from 7.1 #4). Determine whether the sets G is a group under the operation *.

a. $G = \{2,4,6,8\}$ in $\mathbb{Z}_{10}$, $a*b = ab$ (write out the table)

b. $G = \mathbb{Z}$; $a*b = a - b$

d. $G = \{2^x \mid x \in \mathbb{Z}\}$; $a*b = ab$

2. (Adapted from 7.1 # 8) $U_{20} = \{n \in \mathbb{Z}_{20} \mid n \text{ is a unit}\}$.

a. List all of the elements in U_{20}

b. Prove that U_{20} with the operation $a*b = ab$ is a group.

3. (similar to 7.2 #2) D_6 is all of the symmetry functions on a regular hexagon. Name the $60°$ counterclockwise rotation r, and the reflection in a vertical line v.

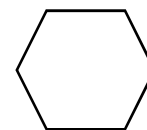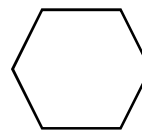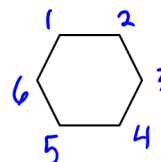a. describe and name all of the other rotations.

b. describe and name all of the other reflections

c. are there any other symmetry functions? If so, describe and name them.

d. Number the vertices to show the result of $v \circ r$

e. Number the vertices to show the result of $r \circ v$

f. List all of the cyclic subgroups of D_6

4. (similar to 7.2 # 3) S_5 is the permutations of 5 elements: 1, 2, 3, 4, 5.

List all of the elements of the cyclic subgroup $\left\langle \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix} \right\rangle$

5. List or describe the elements in (similar to 7.2 # 4-8)

a. $<2>$ in the additive group $\mathbb{Z}_{10}$

b. $<2>$ in the additive group $\mathbb{Z}$

c. $<2>$ in the multiplicative group of non-zero elements $\mathbb{Z}_{11}$ *

6. Prove that $<2>$ in the additive group $\mathbb{Z}_{10}$ is a subgroup.

Also 7.4 # 1 and 4.

Things you should be able to prove about groups:
Theorems 7.1, 7.2, 7.5, 7.6, 7.14