

Suppose:  $h(a) = h(b)$ ;  $a, b \in h$

so  $g \circ f(a) = g \circ f(b)$

and  $g(f(a)) = g(f(b))$

$f(a) = f(b)$  because  $g$   
is 1-to-1

$a = b$  because  $f$  is  
1-to-1

so  $h$  is 1-to-1

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Let:  $a \in T$

then  $x \in S$  and  $g(x) = a$   
 $x$  exists because  $g$  is onto

so  $y \in R$  and  $f(y) = x$  exists  
because  $f$  is onto

and  $h(y) = g(f(y)) = g(x) = a$

so  $h$  is onto.

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Let  $f: R \rightarrow S$  and  $g: S \rightarrow T$

and  $h = g \circ f: R \rightarrow T$

if  $f$  and  $g$  are homomorphisms

prove  $h$  is a homomorphism.

Study for quiz/test March 30

(or is not)

- Prove a set is a sub-ring by checking 4 ring properties (see thm 3.2)

Examples: 3.1 Exercises 1, 5, 6, 8, 11a, 12, 13

3.2 exercises 7, 8, 10

- Prove a ring property

Examples: 3.1 Exercises #22, 23, 24, 25.

~~Know~~ Sample problem: for the set with addition & multiplication in #24, prove that the distributive law is satisfied (Ring axiom 8).

- Prove a solution exists and is unique

Examples: 3.2 Exercises  
5, 12

- Prove a sub-part of theorem 3.5