**Homework on Images, Kernels and Isomorphisms**

**Theorem 53/78:** If *R* and *S* are rings, and  is a ring homomorphism, then is a subring of *S*.

*Proof*:

*first: show that is closed under addition*

Let 

then 

and  because *f* is a homomorphism,

and , so 

So 

*second: show has  additive inverses*

Let 

then  and therefore , so 

by theorem 77, we know , so 

*third: show is closed under multiplication*

***1. Finish the proof of theorem 53/78 by proving the third part****:*

***show is closed under multiplication (you do not need to rewrite parts 1 and 2)***

**Theorem 79:** If *R* and *S* are rings, and  is a ring homomorphism, then  is an ideal in *R*.

*Proof:*

*first: prove is closed under addition*

Let 

then 

And  because *f* is a homomorphism

Therefore 

so 

*second: prove includes additive inverses*

Let 

 and  by theorem 77

So 

Therefore 

***2. Finish the proof of theorem 79 by showing the third part****:*

***prove that multiplicatively absorbs elements of R***

***(you do not need to rewrite parts 1 and 2)***

3. In the proof of theorem 81, why was theorem 58 important? What did it help us to prove?

4. Use Theorem 82 to write down an isomorphism between  and a subset of .