**Permutations in cyclic notation practice:**

Notes:

* A cycle tells what elements cycle-around to each other in the permutation, so (3 1 4) means 3→1 and 1→4 and 4→3. Numbers move left to right through the cycle.
* A cycle can be written in more than one way. (1 4 3) means the same thing as (3 1 4) because all of the numbers get moved to the same places. (1 4 3) is a more standard notation because 1 is the smallest number in the cycle.
* A cycle **never** has the same number in it twice: (1 2 3 1 4) is not allowed
* When you write two cycles next to each other, that means you are composing the permutations. Always do the cycle on the right before the cycle on the left.
* If two cycles have no numbers in common, then you can’t simplify the notation any more. Examples:
  + (1 3 4) ( 2 3 6) is an example of where you should simplify the permutation by composing the functions because both cycles have a 3 in them.
  + (1 3 4) (2 5 6) is already in simplest form because the cycles have no numbers in common.
  + If two cycles have nothing in common then it doesn’t really matter which order you write them in: (1 3 4)(2 5 6)=(2 5 6)(1 3 4). Starting with the smaller first number is more standard, so (1 3 4) (2 5 6) is the more standard notation.

**Practice problems**

Simplify each by composing the cycles:

1. (1 3 4) ( 2 3 6)
2. (1 2 3) (2 4)
3. (1 2)(3 4) (1 4) (2 3)
4. (2 4) (1 2 3)
5. (1 2) (3 4) (1 2 3 4)
6. (1 2 3) (1 4 2 3)
7. (1 2) (1 2 3)
8. (1 2 3) (1 2)
9. (1 2 3 4) (1 3 4)
10. (1 4 2)(1 3)(2 4)

**Worked solutions to problems** 1, 2, 3

1. (1 3 4) ( 2 3 6)

Look right to left. The first place 1 occurs is in the left cycle. 1→3 (1 3…)

Look for 3 (right to left), The first place 3 occurs is in the right cycle. 3→6. 6 is not in the left cycle so 3→6 (1 3 6 …

Look for 6. The first place 6 occurs is in the right cycle. 6→2. 2 is not in the left cycle so 6→2 (1 3 6 2…

Look for 2. A 2 is in the right cycle: 2→3. Look for 3 in the left cycle: 3→4 so 2→3→4. After both functions 2→4 so (1 3 6 2 4…

Look for 4. A 4 is in the left cycle. 4→1. 1 is at the beginning of the cycle, so the cycle is (1 3 6 2 4)

2. (1 2 3) (2 4)

Look for 1: A 1 is in the left cycle, so 1→2. (1 2…

Look for 2. Start looking in the right cycle. The right cycle maps 2→4. The left cycle does not change 4 so 2→ 4 so (1 2 4…

Look for 4. There is a 4 in the right cycle. 4→2. Does the left cycle change 2? Yes. 2→3, so the composition does 4→2→3 and in all 4→3. So (1 2 4 3

Look for 3. The only 3 is in the left cycle, so 3→1, so the answer is (1 2 4 3)

3. (1 2)(3 4) (1 4) (2 3)

Look for 1, start from the right. The first 1 you find is in the cycle (1 4) so the first thing that happens is 1→4. Now look for a 4 in the cycles to the left. There is a 4 in the cycle (3 4) so 1→4→3. Now look for 3 in the cycles to the left, and there are none, so after doing all of the cycles we get 1→3 so (1 3…

Look for a 3, start from the right. The first 3 you find is in the cycle (2 3), so the first thing that happens is 3→2. Now look for a 2 in the cycles to the left. There is a 2 in the cycle (1 2) so 3→2→1. That’s the last cycle going left, so after doing all of the cycles we get 3→1 so (1 3)

Next look for 2. The first 2 (starting on the right) is in the cycle (2 3) so 2→3. Now look in the cycles to the left for a 3. The next 3 you find is in the cycle (3 4) so 2→3→4. Look in the cycles to the left of (3 4) to see if there is another 4, but there are none, so we have (1 3) (2 4…

We expect 4→2: let’s check. The first 4 we find is in (1 4) so 4→1. Look in the cycles to the left for a 1. We find a 1 in the cycle (1 2) so 4→1→2. That’s the left-most cycle so we are done, and yes, 3→2, so we get: (1 3) (2 4)

Answers to 4-10:

1. (2 4) (1 2 3) = (1 4 2 3)
2. (1 2) (3 4) (1 2 3 4) = (2 4)
3. (1 2 3) (1 4 2 3) = (1 4 3 2)
4. (1 2) (1 2 3) = (2 3)
5. (1 2 3) (1 2) = (1 3)
6. (1 2 3 4) (1 3 4) = (1 4 2 3)
7. (1 4 2)(1 3)(2 4) = (1 3 4)