Given $S = \langle r \rangle \leq D_4$ where r is the $90°$ rotation.

Note: $r^1 = r^5$
$90°$ rotation $= 450°$ rotation

Rotations are equal if they differ by a multiple of $360°$

$r^n = r^m$ if and only if

$90 \cdot m - 90n = 360j$ for some $j \in \mathbb{Z}$.

$f: S \to \mathbb{Z}_4$ such that $f(r^n) = [n]_4 \in \mathbb{Z}_4$

☆ If a rotation has two representations:

$r^n = r^m$

then $90m - 90n = 360j$ where $j \in \mathbb{Z}$

so $\dfrac{90m - 90n}{90} = \dfrac{360j}{90}$

$m - n = 4j$

We know $f(r^n) = [n]_4$

$f(r^m) = [m]_4$

but $m - n = 4j$ so $m \equiv n \pmod 4$

so $[n]_4 = [m]_4$

so if $r^n = r^m$

then $f(r^n) = f(r^m)$

so <u>f is a function</u>

$g : S \to \mathbb{Z}$

$g(r^n) = n$

is **not** a function because

$r^1 = r^5$

but $g(r^1) = 1 \neq 5 = g(r^5)$

so $g : r^1 \begin{array}{l} \to 1 \\ \searrow 5 \end{array}$ **not** a function

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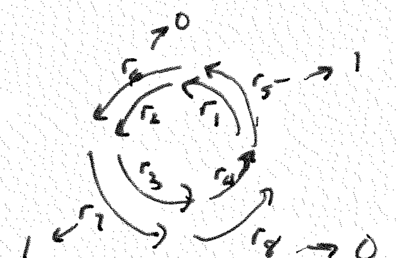
$h : S \to \mathbb{Z}_2$

$h(r^n) = [n]_2$

Visualize



equal rotations go to the same thing.

proof: using ☆ if $r^n = r^m$ then

$m - n = 4j$ for some $j \in \mathbb{Z}$.

$h(r^n) = [n]_2$

$h(r^m) = [m]_2$

$m - n = 2(2j)$ so $m \equiv n \mod 2$

so $h(r^n) = h(r^m)$

so h is a function.

$k: S \to \mathbb{Z}_3$

$r^3 = r^7$

and $k(r^3) = [3]_3 = [0]_3$

$k(r^7) = [7]_3 = [1]_3$

$[0] \neq [1]$ in $\mathbb{Z}_3$

so
$$\begin{array}{l} r_3 = r^7 \longrightarrow 0 \\ \qquad\quad \searrow 1 \end{array}$$

k is not a function

<u>Homework</u>: Let $H = \langle r \rangle \leq D_6$ where r is the $60°$ rotation (counter clockwise)

$f: H \to \mathbb{Z}_6$
$f(r^n) = [n]_6$

$g: H \longrightarrow \mathbb{Z}_3$
$g(r^n) = [n]_3$

$h: H \to \mathbb{Z}_5$
$h(r^n) = [n]_5$

For each rule: prove it is a function or prove it is not a function