1. Fill out the operation table for the permutation group $S_3$

| $f \circ g$ do first $(g) \rightarrow$ do second $(f) \downarrow$ | $\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$ | $\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$ | $\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$ | $\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ | $\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$ | $\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$ |
|---|---|---|---|---|---|---|
| $\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$ | | | | | | |
| $\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$ | | | | | | |
| $\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$ | | | | | | |
| $\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ | | | | | | |
| $\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$ | | | | | | |
| $\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$ | | | | | | |

a. Is $S_3$ abelian? Give an example of this from your table.

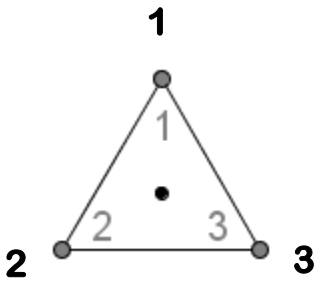
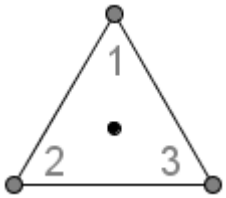
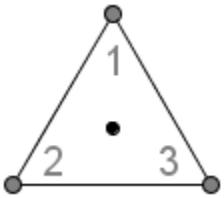
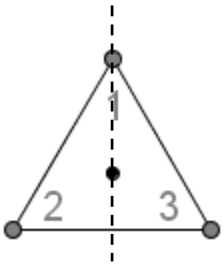
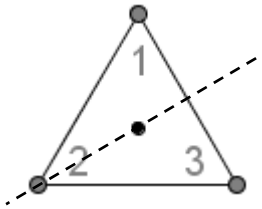
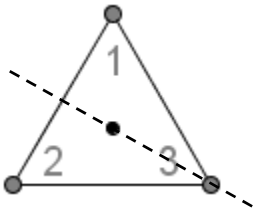
b. What is the identity element for $S_3$?

c. List the inverses of each of these elements:

i. $\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}^{-1} = \begin{pmatrix}1&2&3\\&&\end{pmatrix}$ ii. $\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}^{-1} = \begin{pmatrix}1&2&3\\&&\end{pmatrix}$

iii. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ & & \end{pmatrix}$ iv. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ & & \end{pmatrix}$

2. Write in where each vertex ends after the transformation shown

| | | |
|---|---|---|
| $e$ = identity map<br>rotate 0°<br>  | $r_1$ = rotate 120°<br>counterclockwise<br>  | $r_2$ = rotate 240°<br>counterclockwise<br>  |
| $v$ = reflect in the vertical line<br>  | $u$ = reflect in the line shown<br>  | $w$ = reflect in the line shown<br>  |

Fill out the operation table for the dihedral group $D_3$ of rigid transformations of the equilateral triangle

| $f \circ g$<br>do first $(g) \rightarrow$<br>do second $(f) \downarrow$ | $e$ | $r_1$ | $r_2$ | $v$ | $u$ | $w$ |
|---|---|---|---|---|---|---|
| $e$ | | | | | | |
| $r_1$ | | | | | | |
| $r_2$ | | | | | | |
| $v$ | | | | | | |
| $u$ | | | | | | |
| $w$ | | | | | | |

Is $D_3$ abelian? How do you know?

What is the inverse of each element?

$e^{-1} =$ $v^{-1} =$

$r_1^{-1} =$ $u^{-1} =$

$r_2^{-1} =$ $w^{-1} =$