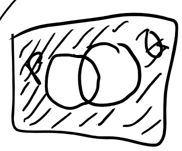
More practice with logic and proofs:

1. Turn these logic statements into set statements, and make both the truth table and the Venn diagram:

a. $\sim(p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

$\overline{P \cup Q}$



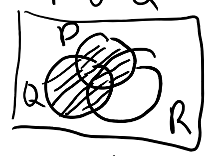
$P \cup Q \rightarrow \overline{P \cup Q}$

b. $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

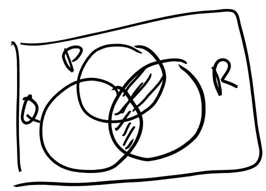
$(P \cup Q) \cap R$

$P \cup Q$



$\downarrow$

$(P \cup Q) \cap R$



2. Use truth tables to show that the statement $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

3. Use truth tables to show that these statements are logically equivalent:

$\sim(p \wedge \sim q)$ and $p \rightarrow q$

p	q	$p \rightarrow q$	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

4. Use truth tables to show these statements are not logically equivalent:

$p \to q$ and $\sim p \to \sim q$

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

5. Write the contrapositive of each of these statements:

a. If n is a multiple of 6, then it is a multiple of 3.

If *n* is not a multiple of 3, then it is not a multiple of 6.

b. If n is a multiple of 3 and a multiple of 2 then it is a multiple of 6

If *n* is not a multiple of 6, then it is not a multiple of 3 or it is not a multiple of 2.

c. If xy is a multiple of 3 then x is a multiple of 3 or y is a multiple of 3

If *x* is not a multiple of 3 and *y* is not a multiple of 3, then *xy* is not a multiple of 3.

d. If n is greater than 10, then it is not a negative number.

If *n* is a negative number then it is less than or equal to 10.

6. Write proofs for each of these statements:

a. If a number is the sum of an even number and an odd number, then it is an odd number.

Given that the number is the sum of an even number and an odd number, then it is equal to $2n+(2m+1)$ for some integers *n* and *m*.

$2n+(2m+1)=2n+2m+1=2(n+m)+1$ so it is odd.

b. If $xy+2y$ is odd then x is odd or y is odd

Suppose *x* is even and *y* is even

Then $x = 2n$ and $y = 2m$ for some integers *n* and *m*,

so $xy + 2y = (2n)(2m) + 2(2m) = 4nm + 4m = 2(2n + 2m)$ is even.

Thus if $xy + 2y$ is odd, then *x* is odd or *y* is odd

c. If $xy > 25$ then $x > 5$ or $y > 5$

If $x \leq 5$ and $y \leq 5$, then $xy \leq 5 \times 5 = 25$

So, if $xy > 25$ then $x > 5$ or $y > 5$.

d. If n is an integer then $n^2 + 3n$ is even.

If n is an integer, then n is even or it is odd.

If n is even, then $n = 2m$ for some integer m. So $(2n)^2 + 3(2n) = 4n^2 + 6n = 2(2n^2+3n)$ is even	If n is even, then $n = 2m+1$ for some integer m. So $(2n+1)^2 + 3(2n+1) =$ $4n^2 +4n+1+ 6n+3 =$ $4n^2 +10n+4 = 2(2n^2+5n+2)$ is even

So, for any integer n, n^2+3n is even.