Discrete Math Final Exam Study List, Spring 2017:

About half of the exam will be on graphs and trees. For these problems you should know how to:

1. Switch a simple graph (or a simple directed graph) between a dot and line format representation and a matrix representation.

2. Prove two graphs are not isomorphic by (specifically!) describing an invariant that is different for the two graphs.

3. Prove that two graphs are isomorphic by: telling the map (which vertex of the first graph maps to which vertex of the second), and writing the matrix representation for both graphs in corresponding order.

4. Know how many edges K_n has.

5. Know how many edges a tree with *n* vertices has.

6. Find an Euler circuit or path in a graph or explain how to know that the graph does not have an Euler circuit or path.

7. Find a Hamilton circuit in a graph or explain how to know the graph does not have a Hamilton circuit.

8. Find a shortest path in a graph using the breadth-first algorithm (both weighted and unweighted problems are possible).

9. Find and show the chromatic number of a graph.

10. Draw a graph with chromatic number 2, 3, 4 or 5.

11. (Find a directed Euler circuit or path in a directed multi-graph, or explain how we know none exists).

12. (Turn a graph or multigraph into strongly connected directed graph using the depth first search algorithm, or explain how we know none exists).

13. Explain why a graph is or is not:

- a tree
- simple
- multi
- directed

14. Find a spanning tree using the breadth-first algorithm.

15. Find a minimal or maximal spanning tree using Prim's (greedy) algorithm

16. Find a spanning tree using the depth-first algorithm.

About half of the final exam will be on prior content from the course. In particular, I am considering the following problems:

From the first review:

- Using the PERT Algorithm to show a total time and critical path for a task description (practice: do # 14 or 15 on pg 9) or Follow a written algorithm (see 1.4 and also handout Feb 6)
- Finding a number of ways something can be accomplished (order matters) using multiplication or permutations (see 1.2 # 21, 23)
- Simplifying a formula with permutations using the factorial formula (see 1.2 # 30-32)
- Finding the number of subsets of a set (see 1.3)
- Show steps to find a Venn diagram to represent a set. (2.1)

From the second review:

- Decide whether a relation is (or is not) reflexive, symmetric and transitive, and explain your reasoning. (see 2.2 # 1-12)
- Show how you know that two integers are congruent with a particular mod base. (3.1 # 19-16)
- Compute addition and multiplication in a particular mod base. (3.1 # 17-32)
- Compute exponents in a particular mod base (3.1 # 33-36 and 3.3 # 1-4)

From the third review:

1. Prove by induction that

$5+8+\ldots+(3n+2)=\frac{n(3n+7)}{2}$ **or** $5+15+45+\ldots+5\cdot 3^{n-1}=\frac{5}{2}(3^n-1)$

2. How many 4 element subsets of {a, b, c, d, e, f} are there?

3. Pigeonhole principle:

a. There are 90 widgets that need to be assembled by 8 workers. What is the smallest number that the most efficient worker (the one who assembles the most widgets) might assemble?

b. There are 4 different toys in different cereal boxes. How many boxes do I need to buy to be sure I will have 3 identical toys?

4. There are 5 flavors of Jolly Ranchers. If I grab 10 Jolly Ranchers at random out of a bowl, how many different combinations could I get?

5. I have a stack of 15 different Pokemon cards. 7 are water type and 8 are fire type. Assume each has a different number of HP.

a. In how many ways can I choose 5 cards?

b. In how many ways can I choose 3 water type and 2 fire type cards?

c. If I choose 5 cards at random, what is the probability that 3 are water type and 2 are fire type?

d. If I put down 5 cards, one at a time, how many orders are there?

e. If I put down 5 cards in a row, what is the probability that the first card has the highest HP?

f. If I put down 5 cards in a row, what is the probability that they are in order of decreasing HP? **1/5!**

7. a. I am going to give 15 different Pokemon cards to 4 cub scouts, so scout A gets 6 cards, scout B gets 4 cards, scout C gets 3 cards and scout D gets 2 cards. How many different possible outcomes are there?

b. What is the probability that scout D gets the cards with the two highest HP?

From the logic and proof review: (expect at least one truth table problem and at least one proof)

1. Turn these logic statements into set statements, and make both the truth table and the Venn diagram:

a. $\sim(p\vee q)$ b. $(p\vee q)\wedge r$

2. Use truth tables to show that the statement $[(p\to q)\wedge(q\to r)]\to(p\to r)$ is a tautology

3. Use truth tables to show that these statements are logically equivalent: $\sim(p\wedge\sim q)$ and $p\to q$

4. Use truth tables to show these statements are not logically equivalent: $p\to q$ and $\sim p\to\sim q$

6. Write proofs for each of these statements:

a. If a number is the sum of an even number and an odd number, then it is an odd number.

b. If $xy+2y$ is odd then x is odd or y is odd

c. If $xy>25$ then $x>5$ or $y>5$

d. If n is an integer then n^2+3n is even.

From the chapter 9 review:

1. Prove by induction that $5\cdot 3^n-3$ is an explicit formula for the recursively defined function: $S_n=3S_{n-1}+6$ where $S_0=2$

2. Find an explicit formula for the function. Show your work. $S_n=5S_{n-1}+3$ where $S_0=4$