Discrete Math Final Exam Study List, Spring 2018:

About half of the exam will be on graphs and trees. For these problems you should know how to:

1. Switch a simple graph (or a simple directed graph) between a **dot and line** format representation and a **matrix** representation.

2. For a pair of graphs, be able to do the correct one of the following:

- Prove two graphs are not isomorphic by (specifically!) describing an invariant that is different for the two graphs.
- Prove that two graphs are isomorphic by: telling the map (which vertex of the first graph maps to which vertex of the second).

3. Know how many edges K_n has.

4. Know how many edges a tree with *n* vertices has.

5. For a given graph, be able to do the correct one of the following:

- Find an Euler circuit
- Find an Euler path
- Explain why the graph does not have an Euler circuit or path (using vertex degrees)

6. For a given graph, be able to do the correct one of the following:

- Find a Hamiltonian circuit
- Prove that the graph does not have a Hamiltonian circuit.

7. Find a shortest path/the shortest distances from a given start in a graph (both weighted and unweighted problems are possible).

8. Find and show the chromatic number of a graph.

9. Turn a map into a graph, and find a map coloring

10. Draw a tree that would fit a given chemical formula (C_3H_8)

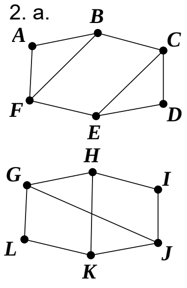
11. Find a spanning tree using the breadth-first algorithm.

12. Find a spanning tree using the depth-first algorithm.

13. Find a minimal or maximal spanning tree using Prim's (greedy) algorithm

14. Turn a graph or multigraph into strongly connected directed graph using the depth first search algorithm, or explain how we know none exists.

2. a.



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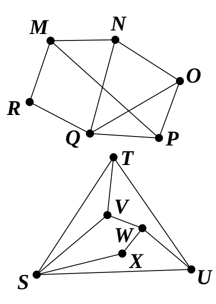

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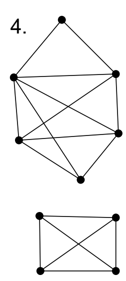

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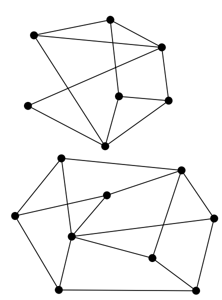


About half of the final exam will be on prior content from the course. In particular, I am considering the problems like the following

From the first review:

15. Sketch the Venn diagram to show the following set relationships.

a. $(A\cup B)\cap C$ b. $A\cup\overline{B}$ c. $\overline{A\cap B}$ d. $A\cap(\overline{B\cup C})$

16. Make truth tables for each of these, and tell which, if any, are equivalent:

a. $p\to(q\wedge r)$ b. $(\sim q)\wedge(\sim r)\to\sim p$ c. $\sim(p\wedge(\sim(q\wedge r))$ d. $((\sim q)\vee(\sim r))\wedge p$

17-18: for each statement, circle the equivalent statements:

17. If it is a square, then it is a rectangle.

a. All squares are rectangles
b. All rectangles are squares
c. Some squares are rectangles
d. All non-rectangles are non-squares
e. All non-squares are non-rectangles

18. Every convergent sequence is Cauchy and bounded

a. If a sequence is Cauchy then it is convergent and bounded
b. If a sequence is convergent then it is Cauchy and bounded
c. If a sequence is not convergent then it is not Cauchy or not bounded.
d. If a sequence is not Cauchy or not bounded then it is not convergent
e. If a sequence is both not Cauchy and not bounded then it is not convergent
f. If a sequence is not both Cauchy and bounded then it is not convergent

19. List an element of each of these sets

a. $C^1(\mathbb{R})=\{f:\mathbb{R}\to\mathbb{R}\mid f\text{ is differentiable}\}$ b. $C^1(\mathbb{R})\times\mathbb{Z}$

c. $2^{\mathbb{Z}}$ d. $E=\{\{(x,y)\in\mathbb{R}^2\mid(ax)^2+(by)^2=1\}\mid a,b\in\mathbb{R}\}$

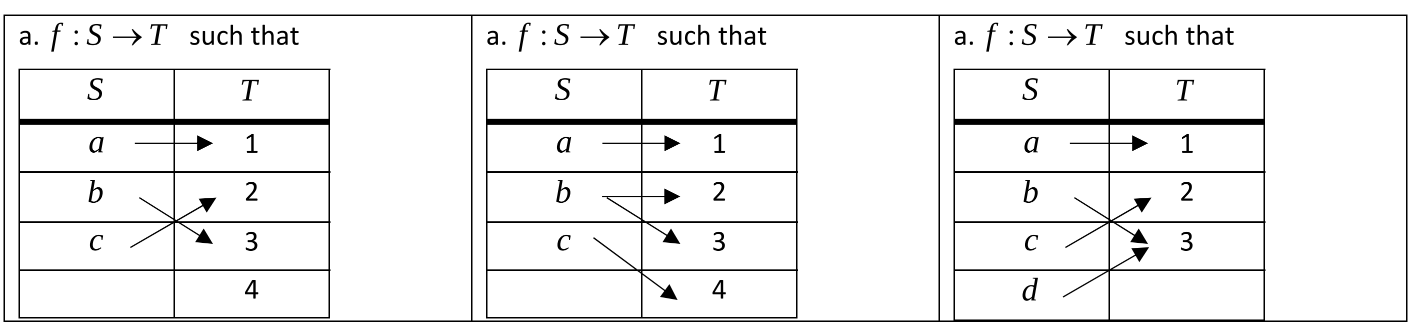
20. Tell a domain and a codomain that would make sense for each function:

a. $f(x,y,z)=(x,y+z)$ b. . $g(a+bi)=b$ c. $F(f(x),a)=f'(a)$

21. For the function $f:\mathbb{R}^2\to\mathbb{R}^2$ such that $f(x,y)=(x+y,x^2)$

Find the image: $f(3,4)$ and the pre-image: $f^{\leftarrow}(5,4)$

22. For each of the following, tell if it is a. a function, b. one-to-one, c. onto



23. Compute: a. $4 \times 5 \pmod 7$ b. $2-8 \pmod{10}$ c. $6 \times 4 + 7 \pmod 9$

From the second review: Just the probability and counting problems

24. From the set: {a, b, c, d, e, f}
a. How many subsets does the set have?
b. How many 4-element subsets does the set have?

25. There are 90 widgets that need to be assembled by 8 workers. What is the smallest number that the most efficient worker (the one who assembles the most widgets) could assemble?

26. There are 5 flavors of Jolly Ranchers: Grape, Apple, Watermelon, Cherry and Blue Raspberry
a. If I grab 10 Jolly Ranchers at random out of a bowl, how many different combinations could I get?
b. If I randomly choose 4 Jolly Ranchers, what is the probability that they are all the same flavor?
c. If I randomly choose 4 Jolly Ranchers, what is the probability that they are 4 different flavors?
d. If I give Jolly ranchers to 4 people (randomly) what is the probability that the first person gets apple, **and** the last 2 people get grape?
e. If I give Jolly ranchers to 4 people (randomly) what is the probability that the first person gets apple, **or** the last 2 people get grape?
f. If I grab some Jolly Ranchers without looking, how many do I need to get to be sure I will have at least 3 of the same flavor?

27. I have a stack of 15 different Pokemon cards. 7 are water type and 8 are fire type. Assume each has a different number of HP (so they can all be distinguished)
a. In how many ways can I choose 5 cards?
b. In how many ways can I choose 3 water type and 2 fire type cards?
c. If I choose 5 cards at random, what is the probability that 3 are water type and 2 are fire type?
d. If I put down 5 cards, one at a time, how many orders are there?
e. If I put down 5 cards in a row, what is the probability that the first card has the highest HP?
f. If I put down 5 cards in a row, what is the probability that they are in order of decreasing HP?

28. a. How many distinguishable rearrangements are there of the word Massachusetts?
b. What is the probability in a rearrangement of Massachusetts that both of the a's will be together, all of the s's will be together and both of the t's will be together?

From the third review: iteration, PERT diagrams and algorithms

- Use iteration to find an explicit formula for a sequence given by an recursive relation. See pg 480 # 11-19. In particular, you are likely to be asked for the explicit formula for a recurrence relation of the form $s_n = a \cdot s_{n-1} + b$ where $a \neq \pm 1$ or of the form $s_n = s_{n-1} + bn + c$
- Use a PERT diagram or a similar strategy to find the projected time to completion for a complex task and the critical path for the task. pg 9 # 9-16
- Tell (as a function) how many elementary operations a given algorithm uses. pg 33 # 27-30, 33, pg 39 # 31, 32