Integration by parts, extra practice/relearn:

Pre-requisite skill: Integrate sinx, cosx and e^x with linear substitutions. The faster and easier these are for you the better.

Practice:

a. $\int \sin 2x\,dx$ b. $\int \sin 3x\,dx$ c. $\int \sin(x/2)\,dx$

d. $\int \cos 2x\,dx$ e. $\int \cos 3x\,dx$ f. $\int \cos(x/2)\,dx$

g. $\int e^{2x}dx$ h. $\int e^{3x}dx$ i. $\int e^{x/2}dx$ j. $\int \frac{1}{e^{2x}}dx$ k. $\int \frac{1}{e^{3x}}dx$ l. $\int \frac{1}{e^{x/2}}dx$ m. $\int \frac{1}{e^{x}}dx$

Basic integral by parts: x times sinx, cosx or e^x

Example pattern:

$\int x\cos(3x)\,dx$

Set up u and dv. For x times sinx, cosx or e^x, u will always be x.

$u = x \quad dv = \cos(3x)\,dx$

$du = 1\cdot dx \quad v = \int \cos(3x)\,dx = \frac{\sin(3x)}{3}$

Take the derivative of u and the integral of dv.
You may want to write out a substitution $w = 3x$
You do not need +C here.

$$\int x\cos(3x)\,dx = x\frac{\sin(3x)}{3} - \int \frac{\sin(3x)}{3}\cdot 1\cdot dx = \frac{x\sin(3x)}{3} - \frac{-\cos(3x)}{3\cdot 3} + C = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + C$$

Use the formula $uv - \int v\,du$

Integrate $\int \frac{\sin(3x)}{3}dx$. You may want to write out a substitution $w = 3x$. Here is where you need to add +C

Practice:

1. $\int x\sin 3x\,dx$ 2. $\int x\sin(x/2)\,dx$ 3. $\int x\cos 2x\,dx$ 4. $\int xe^{2x}dx$ 5. $\int \frac{x}{e^{3x}}dx$

6. $\int 5x\sin 2x\,dx$ 7. $\int (x+5)\cos 3x\,dx$ 8. $\int 3x\cos(x/2)\,dx$

9. $\int 2xe^{3x}dx$ 10. $\int (3x+7)e^{x/2}dx$ 11. $\int \frac{3x}{e^{2x}}dx$

Answers:

a. $\frac{-\cos(2x)}{2}$ b. $\frac{-\cos(3x)}{3}$ c. $-2\cos(x/2)$ d. $\frac{\sin(2x)}{2}$ e. $\frac{\sin(3x)}{3}$ f. $2\sin(x/2)$

g. $\frac{e^{2x}}{2}$ h. $\frac{e^{3x}}{3}$ i. $2e^{x/2}$ j. $\frac{e^{-2x}}{-2} = \frac{-1}{2e^{2x}}$ k. $\frac{e^{-3x}}{-3} = \frac{-1}{3e^{3x}}$ l. $-2e^{-x/2} = \frac{-2}{e^{x/2}}$ m. $-e^{-x} = \frac{-1}{e^{x}}$

1. $\frac{-x\cos(3x)}{3} + \frac{\sin(3x)}{9} + C$ 2. $-2x\cos(x/2) + 4\sin(x/2) + C$ 3. $\frac{x\sin 2x}{2} - \frac{\cos 2x}{4} + C$

4. $\frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$ 5. $\frac{-x}{3e^{3x}} - \frac{1}{9e^{3x}} + C$ 6. $\frac{-5x\cos(2x)}{2} + \frac{5\sin(2x)}{4} + C$ 7. $\frac{(x+5)\sin(3x)}{3} - \frac{\cos(3x)}{9} + C$

8. $6\sin(x/2) - 12\cos(x/2) + C$ 9. $\frac{2xe^{3x}}{3} - \frac{2e^{3x}}{9} + C$ 10. $(6x+20)e^{x/2} + C$ 11. $\frac{-3x}{2e^{2x}} - \frac{3}{4e^{2x}} + C$