1. $\int \underbrace{x^2}_{u} \underbrace{\sin(3x)\,dx}_{dv}$

$\int u\,dv = uv - \int v\,du$

$du = 2x\,dx \qquad v = \int \sin(3x)\,dx$

$w = 3x$

$dw = 3dx$

$\frac{1}{3}dw = dx$

$\int \sin w \frac{1}{3} dw$

$= -\frac{\cos w}{3} = \frac{-\cos(3x)}{3}$

$\frac{-x^2\cos 3x}{3} + \int \frac{\cos(3x)}{3} 2x\,dx$

$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\int \underbrace{x}_{u}\underbrace{\cos(3x)\,dx}_{dv} \qquad du = dx \qquad \int \cos(3x)\,dx$

$= \frac{1}{3}\int \cos w\,dw$

$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} - \int \frac{1}{3}\sin 3x\,dx\right) \qquad v = \frac{1}{3}\sin w = \frac{1}{3}\sin(3x)$

$\frac{-x^2\cos 3x}{3} + \frac{2}{3}\left(\frac{x\sin(3x)}{3} + \frac{1}{3}\,\frac{\cos 3x}{3}\right) + C$

$-\frac{x^2\cos(3x)}{3} + \frac{2x\sin(3x)}{9} + \frac{2\cos(3x)}{27} + C$

2. $\int \sin^2 x \cos^3 x\, dx$

$u = \sin x \qquad du = \cos x\, dx$

$\int \sin^2 x \cos^2 x \cos x\, dx$

$\int \sin^2 x (1 - \sin^2 x) \cos x\, dx$

$\int u^2 (1 - u^2)\, du$

$= \int u^2 - u^4\, du$

$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

3. $\int \underbrace{\sin^{-1} x}_{u}\, \underbrace{dx}_{dv}$

$$\int u\,dv = uv - \int v\,du$$

$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$dv = dx$$
$$v = x$$

$$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$u = 1-x^2$$
$$du = -2x\,dx$$
$$-\frac{1}{2}du = x\,dx$$

$$= x\sin^{-1}x - \int \frac{1}{\sqrt{u}}\left(-\frac{1}{2}\right)du$$

$$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du = x\sin^{-1}x + \frac{1}{\cancel{2}}\,\frac{u^{1/2}}{\cancel{1/2}} + C$$

$$= x\sin^{-1}x + \sqrt{1-x^2} + C$$

4. $\int \frac{2x^2+3x-8}{(x-4)(x+2)^2}\,dx$

$$\cancel{(x-4)(x+2)^2}\,\frac{2x^2+3x-8}{\cancel{(x-4)(x+2)^2}} = \frac{A\,\cancel{(x-4)}(x+2)^2}{\cancel{x-4}} + \frac{B\,(x-4)\cancel{(x+2)}(x+2)}{\cancel{(x+2)}} + \frac{C\,(x-4)\cancel{(x+2)^2}}{\cancel{(x+2)^2}}$$

$$2x^2+3x-8 = Ax^2+4Ax+4A + Bx^2-2Bx-8B+Cx-4C$$

$$2 = A+B$$
$$3 = 4A-2B+C \xrightarrow{\times 4} 12 = 16A-8B+4C$$
$$-8 = 4A-8B-4C \qquad -8 = 4A-8B-4C$$

$$4 = 20A-16B$$
$$\xrightarrow{\times 16}\ 32 = 16A+16B$$
$$36 = 36A \rightarrow A=1$$

$$2 = 1+B \qquad 3 = 4-2+C$$
$$1 = B \qquad 3 = 1+C \qquad C=1$$

$$= \int \frac{1}{x-4}\,dx + \int \frac{1}{x+2}\,dx + \int \frac{1}{(x+2)^2}\,dx$$

$du = dx \quad u = x-4 \qquad v = x+2 \quad dv = dx$

$$= \int \frac{1}{u}\,du + \int \frac{1}{v}\,dv + \int \frac{1}{v^2}\,dv \qquad \frac{1}{v^2} = v^{-2}$$

$$= \ln|u| + \ln|v| + \frac{v^{-1}}{-1} + C$$

$$= \ln|x-4| + \ln|x+2| - (x+2)^{-1} + C$$

5. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= \int \frac{1}{u} (-1) \, du$

$= -\ln|u| + C = \boxed{-\ln|\cos x| + C}$

$= \ln|\sec x|$

6. $\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv}$

$$\int u\,dv = uv - \int v\,du$$

$dv = dx$

$v = x$

$u = \ln x$

$du = \frac{1}{x}dx$

$$x\ln x - \int x \frac{1}{x}\,dx$$

$$x\ln x - \int 1\,dx$$

$$x\ln x - x + C$$

7. $\int \frac{4x^2+5x}{(x-1)(x^2+2)}\,dx$

$$\frac{4x^2+5x}{(x-1)(x^2+2)} = \frac{A}{(x-1)} + \frac{(Bx+C}{x^2+2}$$

$(x-1)(x^2+2)$ $\quad$ $(x-1)(x^2+2)$ $\quad$ $(x-1)(x^2+2)$

$$4x^2+5x = Ax^2+2A+Bx^2-Bx+Cx-C$$

$4 = A+B$

$5 = -B+C$

$0 = 2A-C$

$5 = 2A-B$

$4 = A+B$

$5 = 2A-B$

$9 = 3A$

$A=3$

$5=-1+C$

$6=C$

$4=3+B \to B=1$

$$\int \frac{3}{x-1}dx + \int \frac{x+6}{x^2+2}dx = \int \frac{3}{x-1}dx + \int \frac{x}{x^2+2}dx + \int \frac{6}{x^2+2}dx$$

$u=x-1$ $\quad$ $v=x^2+2$ $\quad$ $x^2+2=2w^2+2$

$du=dx$ $\quad$ $dv=2x\,dx$ $\quad$ $x=\sqrt{2}\,w$

$\frac{1}{2}dv = x\,dx$ $\quad$ $dx=\sqrt{2}\,dw$

$w=\frac{x}{\sqrt{2}}$

$$\int \frac{3}{x-1}dx + \int \frac{1}{v}\frac{1}{2}dv + \int \frac{6}{(\sqrt{2}w)^2+2}\sqrt{2}\,dw$$

$$\int \frac{3}{u}du + \frac{1}{2}\ln|v| + \int \frac{6\sqrt{2}}{2w^2+2}dw$$

$$= 3\ln|u| + {''} + \frac{\overset{3}{\cancel{6}}\sqrt{2}}{\cancel{2}}\int \frac{1}{w^2+1}dw$$

$$= 3\ln|x-1| + \frac{1}{2}\ln|x^2+2| + 3\sqrt{2}\tan^{-1}(w)$$

$$= 3\ln|x-1| + \frac{1}{2}\ln|x^2+2| + 3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$