

A problem like #1

$$\int \frac{dx}{x^2 \sqrt{9-25x^2}}$$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$9 - 9\sin^2 \theta = 9\cos^2 \theta$$

$$9 - 25x^2$$

$$25x^2 = 9\sin^2 \theta$$

$$\sqrt{25x^2} = \sqrt{9\sin^2 \theta}$$

$$5x = 3\sin \theta$$

$$x = \frac{3}{5} \sin \theta$$

$$dx = \frac{3}{5} \cos \theta d\theta$$

$\swarrow \frac{dx}{d\theta}$

$$= \int \frac{\frac{3}{5} \cos \theta d\theta}{\left(\frac{3}{5} \sin \theta\right)^2 \sqrt{9-25\left(\frac{3}{5} \sin \theta\right)^2}}$$

$$= \int \frac{\frac{3}{5} \cos \theta d\theta}{\frac{9}{25} \sin^2 \theta \sqrt{9-25 \cdot \frac{9}{25} \sin^2 \theta}} = \int \frac{3 \cdot 25 \cos \theta d\theta}{5 \cdot 9 \sin^2 \theta \sqrt{9(1-\sin^2 \theta)}}$$

$$= \int \frac{5 \cos \theta d\theta}{3 \sin^2 \theta \sqrt{9} \sqrt{\cos^2 \theta}} = \int \frac{5 \cos \theta d\theta}{3 \sin^2 \theta \cdot 3 \cos \theta}$$

$$= \frac{5}{9} \int \frac{1}{\sin^2 \theta} d\theta = \frac{5}{9} \int \csc^2 \theta d\theta = \frac{5}{9} (-\cot \theta) + C$$

$$x = \frac{3}{5} \sin \theta$$

$$\frac{5}{3} x = \sin \theta = \frac{\text{op}}{\text{hyp}}$$



$$9^2 + (5x)^2 = 9^2$$

$$a^2 = 9 - 25x^2$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{op}}$$

$$= -\frac{5}{9} \frac{\sqrt{9-25x^2}}{5x} + C$$

$$= -\frac{\sqrt{9-25x^2}}{9x} + C$$