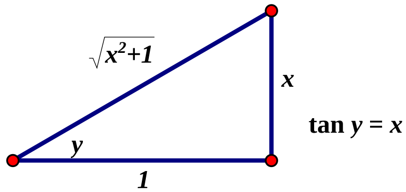
7 derivative problems you should know how to do:

1. Prove that $\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$



$$y=\tan^{-1}x$$
$$\tan y=\tan(\tan^{-1}x)$$
$$\tan y=x$$
$$\sec^2 y\frac{dy}{dx}=1$$

sec y = $\frac{\textbf{hyp}}{\textbf{adj}}=\sqrt{x^2+1}$

$$\frac{dy}{dx}=\frac{1}{\sec^2 y}=\frac{1}{(\sqrt{x^2+1})^2}=\frac{1}{x^2+1}=\frac{1}{1+x^2}$$

2. $\frac{d}{dx}x^{\sec x}$

$$y=x^{\sec x}$$
$$\ln y=\ln x^{\sec x}$$
$$\ln y=\sec x\ln x$$
$$\frac{d}{dx}\ln y=\frac{d}{dx}(\sec x\ln x)$$
$$\frac{1}{y}\frac{dy}{dx}=\sec x\cdot\tan x\cdot\ln x+\sec x\cdot\frac{1}{x}$$
$$\frac{dy}{dx}=y\left(\sec x\cdot\tan x\cdot\ln x+\sec x\cdot\frac{1}{x}\right)$$
$$\frac{dy}{dx}=x^{\sec x}\left(\sec x\cdot\tan x\cdot\ln x+\sec x\cdot\frac{1}{x}\right)$$

3. $\frac{d}{dx}2^x=\frac{d}{dx}e^{\ln 2^x}=\frac{d}{dx}e^{x\ln 2}$

$$e^{x\ln 2}\cdot\ln 2=2^x\cdot\ln 2$$

Alternately, do:

$$y=2^x$$
$$\ln y=\ln 2^x=x\ln 2$$
$$\frac{d}{dx}\ln y=\frac{d}{dx}x\cdot\ln 2$$
$$\frac{1}{y}\frac{dy}{dx}=\ln 2$$
$$\frac{dy}{dx}=y\cdot\ln 2=2^x\cdot\ln 2$$

4. $\frac{d}{dx}\log_5(x^2+3x)=\frac{d}{dx}\frac{\ln(x^2+3x)}{\ln(5)}$

$$=\frac{d}{dx}\frac{1}{\ln 5}\ln(x^3+3x)=\frac{1}{\ln 5}\frac{d}{dx}\ln(x^3+3x)$$
$$=\frac{1}{\ln 5}\frac{1}{x^3+3x}(3x^2+3)=\frac{3x^2+3}{(x^3+3x)\ln 5}$$

5. $\frac{d}{dx}e^{\tan x}+\ln(x^5(3x+7))$

$$=\frac{d}{dx}e^{\tan x}+5\ln x+\ln(3x+7)$$
$$=e^{\tan x}\sec^2 x+5\cdot\frac{1}{x}+\frac{1}{3x+7}\cdot 3$$
$$=e^{\tan x}\sec^2 x+\frac{5}{x}+\frac{3}{3x+7}$$

6. $\frac{d}{dx}\sqrt{x}\,e^{4x}=\frac{d}{dx}x^{1/2}e^{4x}$

$$=\frac{1}{2}x^{-1/2}e^{4x}+x^{1/2}e^{4x}\cdot 4$$
$$=\frac{e^{4x}}{2\sqrt{x}}+4\sqrt{x}\,e^{4x}$$

7. $\frac{d}{dx}\frac{e^{3x}+e^{-3x}}{x^2}$

$$=\frac{(3e^{3x}-3e^{-3x})x^2+(e^{3x}-e^{-3x})2x}{x^4}$$
$$=\frac{x\left(x(3e^{3x}-3e^{-3x})+2(e^{3x}-e^{-3x})\right)}{x^4}$$
$$=\frac{x(3e^{3x}-3e^{-3x})+2(e^{3x}-e^{-3x})}{x^3}$$