Some related rates problems

Report at least 2 decimal places in your answers.

Right triangle problems:

1. The boat: a boat is being pulled to the dock by a winch. The winch on the dock is 3 feet above the level of the water. If the winch pulls the boat in at a rate of .5 ft per second, how fast is the boat moving (horizontally) when it is 8 feet from the dock?

2. The kite, take 1: Suppose you are flying a kite, which is staying a vertical distance of 20 feet from the ground (which is flat), and that you are letting out more string at a rate of 1/4 ft per second, how fast is the kite moving horizontally (or, equivalently, how fast is the kite's shadow moving along the ground) when there is 40 ft of string between you and the kite?

3. The kite, take 2: Suppose you are flying a kite, and that you have let out 40 feet of string, and you are letting our more string at a rate of 1/2 ft per second. The kite's shadow is moving along the ground at a rate of 1/4 ft per second, and you can tell from the shadow that the kite is a horizontal distance of 30 ft from you. Is the kite getting higher or lower? How fast is it moving vertically?

4. I look out my front window and see someone walking past on the sidewalk. If the sidewalk is 36 ft away from me, and the person is 39 feet away from me (this is distance from me, not distance they have walked down the side walk), and the person is walking at a rate of 4ft/sec, how fast are they moving away from me?

5. Ships on the ocean, take 1: There are two ships sailing near each other. At noon, ship A is 100 km due west of ship B. Ship A sails west at a rate of 30 kph, and ship B sails north at a rate of 40 kph. How fast is the distance between them increasing at 3 pm.?

6. Ships on the ocean, take 2: There are two ships sailing near each other. At noon, ship A is 100 km due west of ship B. Ship A sails east at a rate of 30 kph, and ship B sails north at a rate of 40 kph. Is the distance between them increasing or decreasing at 2 pm? How fast is the distance changing?

7. Ships on the ocean, take 3: There are two ships sailing near each other. At noon, ship A is 100 km due west of ship B. Ship A sails south at a rate of 30 kph, and ship B sails north at a rate of 40 kph. How fast is the distance between them increasing at 1:30 pm.?

8. Ladders, ladders and more ladders: I have a 15 ft ladder leaning against the wall. Then I start pulling the bottom of the ladder away from the wall at a rate of .5 ft per second. How fast is the top of the ladder going down when I (holding the bottom of the ladder) am a. 2 ft from the wall? b. 6 feet from the wall? c. 12 ft from the wall? Graph the speed of the tip of the ladder as a function of my distance from the wall. When is the speed greatest? When is it least?

9. At the moment in question, Jack is 12 miles due north of the school, traveling towards it at a speed of 30 miles per hour. Jill is 8 miles due west of the school, traveling towards it at a speed of 50 miles per hour. How fast is the distance between Jack and Jill decreasing?

Similar triangles problems. For each of these problems, draw the two similar triangles to show why you need to use similar triangles. Think about why you can't use the Pythagorean theorem to solve these.

10. Movie cameras: A video camera is mounted on a platform that moves back from a large mural at a rate of 2 ft/sec. When the camera is 3 ft from the mural, it sees a rectangular section of the mural 3 ft wide and 2 ft high. When the camera is 5 feet from the mural,
a. How fast is the height of its view increasing?
b. How fast is the area it sees increasing? (Hint, you will need to use the triangles to get an area formula)

11. Street lights, take 1: A man who is 6 ft tall walks away from a streetlight that is 15 ft high. He walks at a rate of 4 ft per second. How fast is his shadow growing when he is 10 feet from the base of the street light?

12. Street lights, take 2: A woman who is 5 ft tall walks away from a streetlight that is 15 ft high. She walks at a rate of 4 ft per second. How fast is the tip of her shadow moving (away from the street light), when she is 12 feet from the base of the street light?

13. Spotlights: a spotlight, mounted on the ground, 30 ft from a wall is shining on the wall. A man who is 6 ft tall is directly between the spotlight and the wall, and is walking towards the wall at a rate of 2 ft per second. How fast is his shadow shrinking when he is 10 ft from the wall?

Area and Volume problems: For each of these, write down the area or volume formula or formulas that you will need before you substitute in any numbers

14. Puddles, take 1: Water from a leaky bucket is dripping slowly onto the flat floor, making a circular puddle. The water is leaking at a constant rate, so the area of the puddle is growing at a constant rate of 3 in^2 per minute. How fast is the radius of the puddle growing when it (the radius) is 4 inches?

15. Puddles, take 2: Water from a leaky bucket is dripping slowly onto the flat floor, making a circular puddle. The water is leaking at a constant rate, so the area of the puddle is growing at a constant rate of 4 in^2 per minute.
a. How fast is the radius of the puddle growing when the area is 6 in^2?
b. How fast is the diameter of the puddle growing when the area is 6 in^2?
c. How fast is the circumference of the puddle growing when the area is 6 in^2?

16. Ripples in a pond: When you throw a rock in a pond, a circular ripple goes outward from that point. If the radius of the ripple increases at a rate of 6 inches per second, how fast is the area enclosed by the ripple increase when the radius is 20 inches?

17. Balloons: A spherical balloon is inflated at a rate of 5 in^3 per second. I am interested in how fast the balloon itself is stretching while it is being inflated, so I want to know how fast the surface area is increasing when the radius of the balloon is 3 inches. (Hint: this is a little like #15c, which was easier if you had already done #15a. What would an easier problem like this one be?)

18. Snowball: Suppose you bring a 10 cm diameter snowball inside. Then the rate at which it melts is proportional to its surface area, so let's say that the melt rate is $2r^2$ cm^3 per minute. I want to know at what rate its diameter is decreasing when it (the diameter) is 8 cm. I will give you some steps, because this one is a bit confusing:
a. At what rate is the volume decreasing when the diameter is 8 cm?
b. How do you relate volume to radius? At what rate is the radius decreasing?
c. How do you relate radius to diameter? At what rate is the diameter decreasing?

19. Suppose that toilet paper is being unrolled from the roll so that the volume of the paper on the roll decreases at a rate of 2 cm^3/sec. Given that the roll is 12 cm. wide, how fast is the radius of the roll decreasing when the diameter is 12 cm?

Changing angle problems:

20. Car watching: A movie camera is tracking a moving car. The camera is 30 feet from the road. A car is driving down the road at a speed of 35 mph which is about 52 feet per second. The camera pans to follow the car as it drives past. How fast should the camera be turning when the car is approaching the camera at a distance 40 feet down the road from the point closest to the camera?