Integrals practice

481. $\int x^3(x^2+5)^6 \, dx$	491. $\int_0^{\pi/6} \sin x \cos^3 x \, dx$
482. $\int \dfrac{x^2}{\sqrt{x+4}} \, dx$	492. $\int \sec^4 x \tan x \, dx$
483. $\int_2^3 x^2(x-2)^4 \, dx$	493. $\int \sec^2 x \tan^4 x \, dx$
484. $\int_{-1}^3 \dfrac{x}{\sqrt{2x+3}} \, dx$	494. $\int_0^{\pi/2} \sqrt{\sin x} \cos x \, dx$
501. $\int \dfrac{3x}{\sqrt{x}} - \dfrac{5}{x^2} \, dx$	521. $\int 2\csc x \cot x \, dx$
502. $\int \dfrac{3x+1}{\sqrt{x}} \, dx$	522. $\int_1^4 x\sqrt{x} - \dfrac{3}{x^2} \, dx$
503. $\int \csc x \cot x \, dx$	523. $\int \dfrac{2x^5-1}{x^2} \, dx$
504. $\int_{\pi/6}^{\pi/3} \csc^2(2x) \, dx$	524. $\int_{-2}^2 5\sqrt[3]{x} + \dfrac{3}{x^2} \, dx$
505. $\int_1^5 \dfrac{1}{(x-3)^2} \, dx$	525. $\int \dfrac{3x}{(x+2)^5} \, dx$
506. $\int_3^4 \dfrac{x}{(x-2)^3} \, dx$	526. $\int_1^3 \sqrt{4x-3} \, dx$
507. $\int \sin^3 x \cos x \, dx$	527. $\int \sec^6 x \tan x \, dx$
508. $\int x^2\sqrt{x^3+2} \, dx$	528. $\int \dfrac{\cos \sqrt{x}}{\sqrt{x}} \, dx$
509. Find the area bounded by: $x=0$, $x=\pi/2$, $y=\sin x$, $y = \dfrac{\sqrt{3}}{2}$	
511. $\int_0^1 x\sqrt{x} - 3x^4 \, dx$	531. $\int_1^3 \dfrac{6x^4-1}{x^2} \, dx$
512. $\int (x+5)(2x-1) \, dx$	532. $\int -\sec x \tan x \, dx$
513. $\int \csc^2 x \, dx$	533. $\int_0^2 \sqrt{4x+1} \, dx$
514. $\int x\sin(x^2) \, dx$	534. $\int 2x\sqrt{x} - \dfrac{3}{x^3} \, dx$
515. $\int_0^1 3x\sqrt{3x+1} \, dx$	535. $\int_{-4}^4 \dfrac{x}{(x+2)^5} \, dx$
516. $\int_{-2}^2 \dfrac{x}{(x+1)^2} \, dx$	536. $\int x(x+2)^5 \, dx$
517. $\int \sec^2 x \tan^5 x \, dx$	537. $\int \sin x \cos^5 x \, dx$
518. $\int \dfrac{\sec^2\left(x^{-2}\right)}{x^3} \, dx$	538. $\int (x^2+1)\sqrt{x^3+3x} \, dx$

Some answers (typos on my part are possible)

481. $\dfrac{(x^2+5)^8}{16} - \dfrac{5(x^2+5)^7}{14} + C$

482. $\dfrac{2(x+4)^{5/2}}{5} - \dfrac{16(x+4)^{3/2}}{3} + 32(x+4)^{1/2} + C$

483. $\dfrac{169}{105}$

484. $\dfrac{4}{3}$

491. $\dfrac{7}{64}$

492. $\dfrac{\sec^4 x}{4} + C$

493. $\dfrac{\tan^5 x}{5} + C$

494. $\dfrac{2}{3}$

Some solutions

1. $\int \frac{3x}{\sqrt{x}} - \frac{5}{x^2} \, dx = \int 3x^{1/2} - 5x^{-2} \, dx = \frac{3x^{3/2}}{3/2} - \frac{5x^{-1}}{-1} + C = \frac{x^{3/2}}{2} + \frac{5}{x} + C$

2. $\int \frac{3x+1}{\sqrt{x}} \, dx = \int (3x+1)x^{-1/2} \, dx = \int 3x^{1/2} + x^{-1/2} \, dx = \frac{3x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{x^{3/2}}{2} + 2x^{1/2} + C$

3. $\int \csc x \cot x \, dx = -\csc x + C$

4. $\int_{\pi/6}^{\pi/3} \csc^2(2x) \, dx \qquad u = 2x \quad du = 2dx \quad dx = (1/2)du$

$\int_{\pi/3}^{2\pi/3} \csc^2(u) \frac{1}{2} \, du = \frac{1}{2}(-\cot(u)) \Big|_{\pi/3}^{2\pi/3} = \frac{1}{2}(-(-\frac{1}{\sqrt{3}})) - \frac{1}{2}(-(\frac{1}{\sqrt{3}})) = \frac{1}{\sqrt{3}}$

5. $\int_1^5 \frac{1}{(x-3)^2} \, dx$ This integral may not exist, and certainly cannot be solved using the

Fundamental Theorem of Calculus because the function is not continuous at x=3 which is in the interval [1,5]

6. $\int_3^4 \frac{x}{(x-2)^3} \, dx \qquad u = x-2 \quad du = dx \quad x = u+2$

$\int_1^2 \frac{u+2}{(u)^3} \, du = \int_1^2 (u+2)u^{-3} \, du = \int_1^2 u^{-2} + 2u^{-3} \, du = \frac{u^{-1}}{-1} + \frac{2u^{-2}}{-2} \Big|_1^2 = \frac{-1}{u} - \frac{1}{u^2} \Big|_1^2 = \frac{-1}{2} - \frac{1}{2^2} - \left(\frac{-1}{1} - \frac{1}{1^2}\right) = -\frac{3}{4} + 2 = \frac{5}{4}$

7. $\int \sin^3 x \cos x \, dx \quad u = \sin x \quad du = \cos x \, dx \quad \int \sin^3 x \cos x \, dx = \int u^3 \, du = \frac{1}{4}u^4 + C = \frac{1}{4}\sin^4 x + C$

8. $\int x^2 \sqrt{x^3 + 2} \, dx \quad u = x^3 + 2 \quad du = 3x^2 dx \quad \frac{1}{3} du = x^2 dx$

$\int x^2 \sqrt{x^3 + 2} \, dx = \int u^{1/2} \frac{1}{3} \, du = \frac{1}{3/2} u^{3/2} \frac{1}{3} + C = \frac{1}{2} u^{3/2} + C = \frac{1}{2}(x^3 + 2)^{3/2}$