166 chapter 4 review and practice exam solutions. Practice for Thursday's problems:

1. For the equation: $y = x^{2/5}(x-3)$ on the interval [-2,4]

1. $y = x^{2/5}(x-3) = 0$

$x^{2/5} = 0 \quad x - 3 = 0$

$x = 0 \qquad x = 3$ roots;

no vertical asymptotes

$$y = x^{2/5}(x-3) = x^{7/5} - 3x^{2/5}$$

$$y' = \frac{7}{5}x^{2/5} - 3\cdot\frac{2}{5}x^{-3/5} = \frac{x^{3/5}}{x^{3/5}}\cdot\frac{7x^{2/5}}{5} - \frac{6}{5x^{3/5}} = \frac{7x-6}{5x^{3/5}}$$

$7x - 6 = 0 \Rightarrow x = 6/7;$

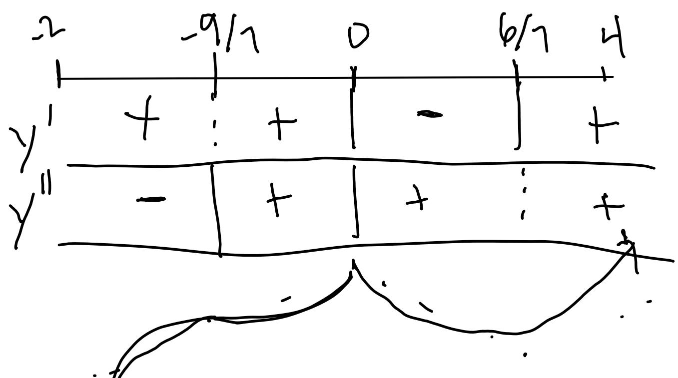
$x^{3/5} = 0 \Rightarrow x = 0$ critical numbers (where the first derivative might change signs)

$$y' = \frac{7}{5}x^{2/5} - 3\cdot\frac{2}{5}x^{-3/5}$$

$$y'' = \frac{7}{5}\cdot\frac{2}{5}x^{-3/5} - 3\cdot\frac{2}{5}\cdot\left(\frac{-3}{5}\right)x^{-8/5} = \frac{14}{25}x^{-3/5} + \frac{18}{25}x^{-8/5} = \frac{x^{5/5}}{x^{5/5}}\cdot\frac{14}{25x^{3/5}} + \frac{18}{25x^{8/5}} = \frac{14x+18}{25x^{8/5}}$$

$14x + 18 = 0 \quad \Rightarrow x = \frac{-9}{7}$

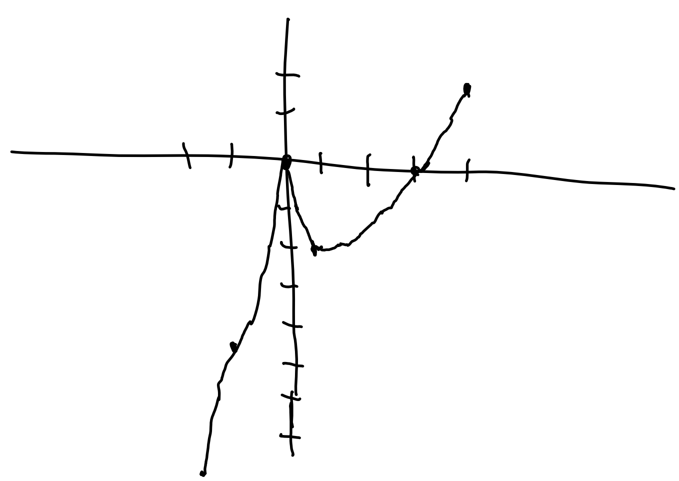
$25x^{8/5} = 0 \quad \Rightarrow x = 0$ numbers where the second derivative might change sign



local max: (0,0) local min: (6/7, -2.01) inflection point: (-9/7, -4.74)

Endpoints are (-2,-6.60), (4, 1.74) so absolute max is (4, 1.74) and absolute min is (-2, -6.60)

graph



2. For the equation $y=\dfrac{x}{(x+3)^2}$

$y=\dfrac{x}{(x+3)^2}$ $\quad x=0 \quad$ root

$(x+3)^2=0 \;\Rightarrow\; x+3=0 \;\Rightarrow\; x=-3$ $\quad$ vertical asymptote

$\lim\limits_{x\to\infty}\dfrac{x}{(x+3)^2}=\lim\limits_{x\to\infty}\dfrac{x}{x^2+6x+9}\cdot\dfrac{1/x^2}{1/x^2}=\lim\limits_{x\to\infty}\dfrac{1/x}{1+6/x+9/x^2}=\dfrac{0}{1}=0$ $\quad y=0$ is a horizontal asymptote

$y'=\dfrac{1\cdot(x+3)^2-x\cdot 2(x+3)}{(x+3)^4}=\dfrac{(x+3)((x+3)-x\cdot 2)}{(x+3)^4}=\dfrac{x+3-2x}{(x+3)^3}=\dfrac{3-x}{(x+3)^3}$

$3-x=0 \;\Rightarrow\; x=3$

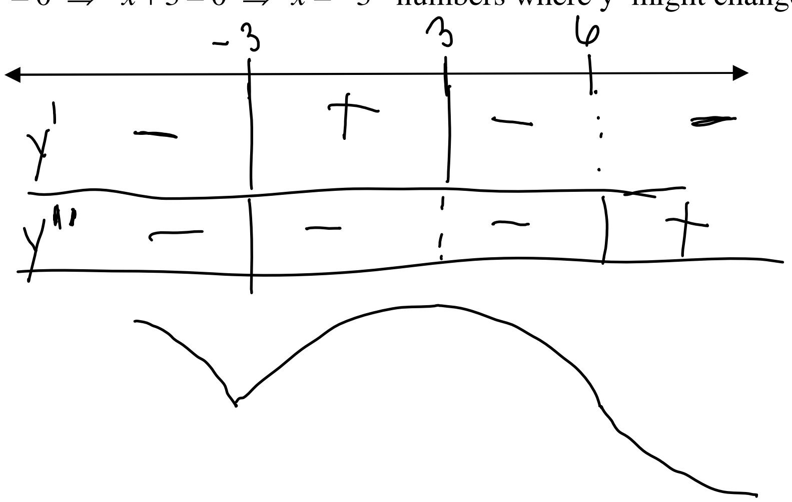
$(x+3)^2=0 \;\Rightarrow\; x+3=0 \;\Rightarrow\; x=-3$ $\quad$ critical numbers (y' might change sign)

$y'=\dfrac{3-x}{(x+3)^3}$

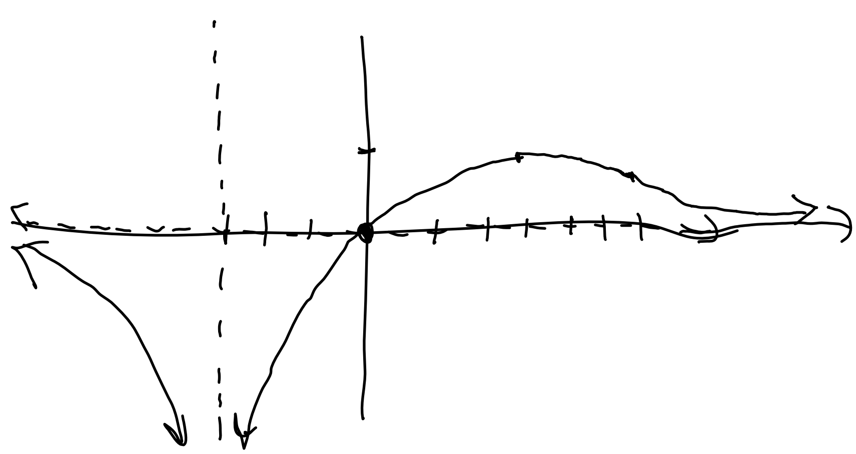
$y''=\dfrac{-1\cdot(x+3)^3-(3-x)3(x+3)^2}{(x+3)^6}=\dfrac{(x+3)^2(-1\cdot(x+3)-(3-x)3)}{(x+3)^6}=\dfrac{-x-3-9+3x}{(x+3)^4}=\dfrac{2x-12}{(x+3)^4}$

$2x-12=0 \;\Rightarrow\; 2x=12 \;\Rightarrow\; x=6$

$(x+3)^4=0 \;\Rightarrow\; x+3=0 \;\Rightarrow\; x=-3$ $\quad$ numbers where y" might change sign



loc min at ~~(-3, undefined)~~ no local min. Local max at (3,0.08), inflection point at (6, 0.074)



3. Find the infinite limits

$$\lim_{x\to\infty}\frac{\sqrt{4+9x^2}}{2x+5}\cdot\frac{1/x}{1/x}=\lim_{x\to\infty}\frac{\sqrt{4+9x^2}}{(2x+5)}\cdot\frac{\sqrt{1/x^2}}{1/x}=\lim_{x\to\infty}\frac{\sqrt{(4+9x^2)1/x^2}}{(2x+5)1/x}=\lim_{x\to\infty}\frac{\sqrt{4/x^2+9}}{2+5/x}=\frac{\sqrt{9}}{2}=\frac{3}{2}$$

$$\lim_{x\to-\infty}\frac{2x+5}{\sqrt{4+9x^2}}\cdot\frac{1/x}{1/x}=\lim_{x\to-\infty}\frac{2x+5}{\sqrt{4+9x^2}}\cdot\frac{1/x}{-\sqrt{1/x^2}}=\lim_{x\to-\infty}\frac{(2x+5)1/x}{\sqrt{(4+9x^2)1/x^2}}\cdot\frac{1}{-1}=\lim_{x\to-\infty}\frac{2+5/x}{-\sqrt{4/x^2+9}}=\frac{-2}{3}$$

$$\lim_{x\to\infty}(\sqrt{4x^2+5x}-2x)\frac{(\sqrt{4x^2+5x}+2x)}{(\sqrt{4x^2+5x}+2x)}=\lim_{x\to\infty}\frac{(4x^2+5x-4x^2)1/x}{(\sqrt{4x^2+5x}+2x)1/x}=\lim_{x\to\infty}\frac{5}{\sqrt{4+5/x}+2}=\frac{5}{4}$$

4. Sketch a graph that satisfies:

$\lim_{x\to 2^-} f(x)=3 \qquad \lim_{x\to 2^+} f(x)=-1$

$f'(x)>0$ for $x<-3, 0<x<2$

$f'(x)<0$ for $-3<x<0$

$f'(x)=0$ for $x=-3$, $x=0$, $x>2$

$f''(x)<0$ for $x<-1.5$

$f''(x)>0$ for $-1.5<x<2$

5. Sketch a continuous graph that satisfies:

$f(0)=1$

$f'(0)$ is undefined $\qquad f'(2)=0$

$f'(x)<0$ for $0<x<2$

$f'(x)>0$ for $x<0$, $x>2$

$f''(x)<0$ for $x>3$

$f''(x)>0$ for $x<0$, $0<x<3$

$\lim_{x\to\infty} f(x)=-1 \qquad \lim_{x\to-\infty} f(x)=0$

	-3	-1.5	0	2		0	2	3
f'	+	-	+	0	f'	+	-	+
f''	-		+		f''	+	+	-
f					f