Review for final exam part 2

Problems 1-5: Prove each max/min/neither statement. You can use the first derivative test, the second derivative test, or show a graph. (In each case you must show that the first derivative is 0 at that point).

1. Prove that $y = x^2 - 4x + 3$ has a local minimum when x=2

2. Prove that $y = x^3 - 6x^2 + 12x - 5$ has a critical point (first derivative is 0) that is neither a max nor a min when x=2

3. Prove that $y = x^4$ has a local minimum when x=0

4. Prove that $y = 2x - e^x$ has a local maximum when x=ln2

5. Prove that $y = xe^x$ has a local minimum when x = -1

Problems 6-12: Practice finding the integrals. Some will need a u-substitution. Some will have numbers to plug in, and some will not.

6. $\int e^{2x} + \frac{3}{x}\, dx$

7. $\int \frac{x}{3} + \frac{2}{x} + \frac{4}{x^2}\, dx$

8. $\int xe^{x^2+3}\, dx$

9. $\int \frac{5}{3x+2}\, dx$

10. $\int_0^4 x\sqrt{2x^2+4}\, dx$

11. $\int_1^2 \frac{5}{x} + \frac{x}{5}\, dx$

12. $\int_0^1 5x + \sqrt{3x+1}\, dx$

Answers

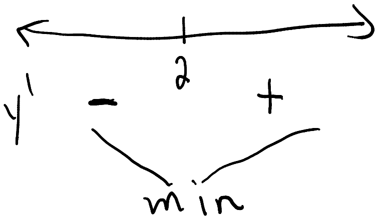
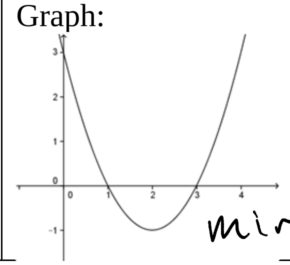
1. Check first derivative:

$y = x^2 - 4x + 3$

$y' = 2x - 4$

$y'(2) = 2 \cdot 2 - 4 = 0$

Then do *one* of the following 3 things:

| First derivative test: | Second derivative test: | Graph: |
|---|---|---|
|  | $y'' = 2 > 0$<br> |  |

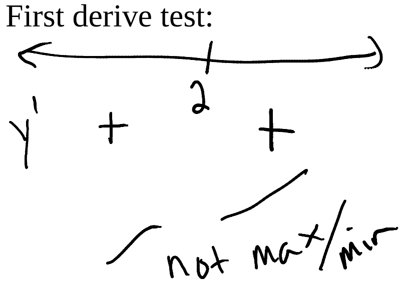
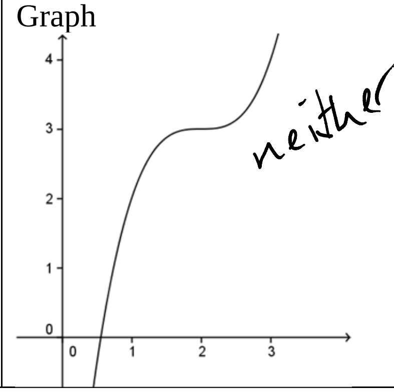
2. Check first derivative:

$y = x^3 - 6x^2 + 12x - 5$

$y' = 3x^2 - 12x + 12$

$y'(2) = 12 - 24 + 12 = 0$

Then do one of the following ~~3~~ 2 things:

| First derive test: | Second derive test: | Graph |
|---|---|---|
|  | $y'' = 6x - 12$<br>$y''(2) = 6 \cdot 2 - 12 = 0$<br>Because second derivative is 0, it doesn't tell us if the point is a max, min or neither |  |

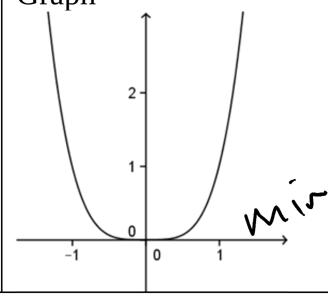
3.Check first derivative:

$y = x^4$

$y' = 4x^3$

$y'(0) = 0$

Then do one of the following ~~3~~ 2 things:

| First derive test: | Second derive test: | Graph |
|---|---|---|
|   | $y'' = 12x^2$<br>$y''(0) = 0$<br>Because second derivative is 0, it doesn't tell us if the point is a max, min or neither |   |

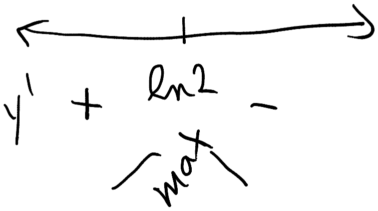
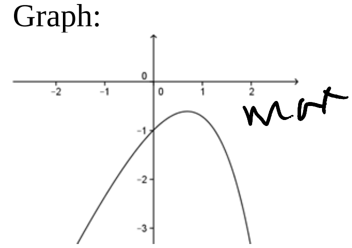
4. Check first derivative:

$y = 2x - e^x$

$y' = 2 - e^x$

$y'(\ln 2) = 2 - e^{ln 2} = 2 - 2 = 0$

Then do *one* of the following 3 things:

| First derivative test: | Second derivative test: | Graph: |
|---|---|---|
|   | $y'' = -e^x$<br>$y''(\ln 2) = -e^{ln 2} = -2 < 0$ ∩<br>max |   |

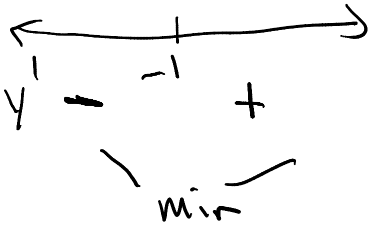
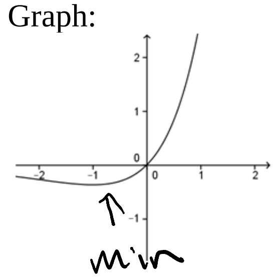
5. Check first derivative:

$y = xe^x$

$y = 1 \cdot e^x + x \cdot e^x$

$y'(-1) = e^{-1} + (-1) \cdot e^{-1} = e^{-1} - e^{-1} = 0$

Then do *one* of the following 3 things:

| First derivative test: | Second derivative test: | Graph: |
|---|---|---|
|   | $y' = e^x + xe^x$<br>$y'' = e^x + 1 \cdot e^x + x \cdot e^x = 2e^x + xe^x$<br>$y''(-1) = 2e^{-1} - e^{-1} = e^{-1} = 1/e > 0$<br>min ∪ |   |

6. $\int e^{2x}+\frac{3}{x}\,dx=\int e^{2x}\,dx+\int \frac{3}{x}\,dx$

$\int e^{2x}\,dx$ $\quad u=2x$ $\quad \int \frac{3}{x}dx$

$=\int e^{u}\cdot\frac{1}{2}du$ $\quad \frac{du}{dx}=2$ $\quad =\int 3\cdot\frac{1}{x}dx$

$=\frac{1}{2}e^{u}+C$ $\quad \frac{1}{2}du=dx$ $\quad =3\ln|x|+C$

$=\frac{1}{2}e^{2x}+C$

$\int e^{2x}+\frac{3}{x}dx=\frac{1}{2}e^{2x}+3\ln|x|+C$

7. $\int \frac{x}{3}+\frac{2}{x}+\frac{4}{x^2}dx=\int \frac{1}{3}\cdot x+2x^{-1}+4x^{-2}\,dx=\frac{1}{3}\cdot\frac{x^2}{2}+2\ln|x|+4\cdot\frac{x^{-1}}{-1}+C=\frac{x^2}{6}+2\ln|x|-\frac{4}{x}+C$

8. $\int xe^{x^2+3}\,dx$

$u=x^2+3$

$\frac{du}{dx}=2x$

$\frac{1}{2x}du=dx$

$\int xe^{x^2+3}dx=\int xe^{u}\cdot\frac{1}{2x}du=\int \frac{1}{2}e^{u}du=\frac{1}{2}e^{u}+C=\frac{e^{x^2+3}}{2}+C$

9. $\int \frac{5}{3x+2}dx$

$u=3x+2$

$\frac{du}{dx}=3$

$\frac{1}{3}du=dx$

$\int \frac{5}{3x+2}dx=\int \frac{5}{u}\cdot\frac{1}{3}du=\int \frac{5}{3}u^{-1}du=\frac{5}{3}\ln|u|+C=\frac{5}{3}\ln|3x+2|+C$

10. $\int_0^4 x\sqrt{2x^2+4}\,dx$

$u = 2x^2 + 4$

$\frac{du}{dx} = 4x$

$\frac{1}{4x}du = dx$

$$\int x\sqrt{2x^2+4}\,dx = \int x\sqrt{u}\cdot\frac{1}{4x}du = \int \frac{1}{4}u^{1/2}\,du = \frac{1}{4}\cdot\frac{u^{3/2}}{(3/2)} + C = \frac{1}{4}\cdot\sqrt{u}^3\cdot\frac{2}{3} + C = \frac{\sqrt{2x^2+4}^3}{6} + C$$

$$\int_0^4 x\sqrt{2x^2+4}\,dx = \left.\frac{\sqrt{2x^2+4}^3}{6}\right|_0^4 = \frac{\sqrt{32+4}^3}{6} - \frac{\sqrt{0+4}^3}{6} = \frac{6^3}{6} - \frac{2^3}{6} = \frac{208}{6} = \frac{104}{3} = 34\frac{2}{3}$$

Alternate notation for #10:

$u = 2x^2 + 4$

$\frac{du}{dx} = 4x$

$\frac{1}{4x}du = dx$

$$\int_0^4 x\sqrt{2x^2+4}\,dx = \int_{x=0}^4 x\sqrt{u}\cdot\frac{1}{4x}du = \int_{x=0}^4 \frac{1}{4}u^{1/2}du = \left.\frac{1}{4}\cdot\frac{u^{3/2}}{(3/2)}\right|_{x=0}^4 = \left.\frac{1}{4}\cdot\sqrt{u}^3\cdot\frac{2}{3}\right|_{x=0}^4$$

$$\left.\frac{\sqrt{2x^2+4}^3}{6}\right|_0^4 = \frac{\sqrt{32+4}^3}{6} - \frac{\sqrt{0+4}^3}{6} = \frac{6^3}{6} - \frac{2^3}{6} = \frac{208}{6} = \frac{104}{3} = 34\frac{2}{3}$$

11. $\int_1^2 \frac{5}{x} + \frac{x}{5}\,dx = \int_1^2 5\cdot\frac{1}{x} + \frac{1}{5}\cdot x\,dx = \left.5\ln|x| + \frac{1}{5}\cdot\frac{x^2}{2}\right|_1^2 = 5\ln 2 + \frac{4}{10} - \left(5\ln 1 + \frac{1}{10}\right) = 5\ln 2 + \frac{3}{10}$

12. $\int_0^1 5x+\sqrt{3x+1}\,dx = \int_0^1 5x\,dx + \int_0^1 \sqrt{3x+1}\,dx$

$$\int_0^1 5x\,dx = 5\cdot\frac{x^2}{2}\Bigg|_0^1 = \frac{5}{2}-0 = \frac{5}{2} = 2\frac{1}{2}$$

$$u = 3x+1 \qquad \Rightarrow \qquad \frac{du}{dx} = 3 \qquad \Rightarrow \qquad \frac{1}{3}du = dx$$

$$\int \sqrt{3x+1}\,dx = \int \sqrt{u}\cdot\frac{1}{3}du = \int \frac{1}{3}u^{1/2}du = \frac{1}{3}\cdot\frac{u^{3/2}}{3/2}+C = \frac{1}{3}\sqrt{u}^3\cdot\frac{2}{3}+C = \frac{2}{9}\sqrt{3x+1}^3+C$$

$$\int_0^1 \sqrt{3x+1}\,dx = \frac{2}{9}\sqrt{3x+1}^3\Bigg|_0^1 = \frac{2}{9}\sqrt{4}^3 - \frac{2}{9}\sqrt{1}^3 = \frac{16}{9}-\frac{2}{9} = \frac{14}{9} = 1\frac{5}{9}$$

$$\int_0^1 5x+\sqrt{3x+1}\,dx = 2\frac{1}{2}+1\frac{5}{9} = 2\frac{9}{18}+1\frac{10}{18} = 3\frac{19}{18} = 4\frac{1}{18}$$